SENIOR DIVISION

Questions 1 - 10, 3 marks each

1. Which of the following is closest to 9?

(A) 9.2 (B) 8.17 (C) 8.7 (D) 9.21 (E) 8.71

2. $5x - 3 - (3 - 5x)$ equals

(A) 0 (B) $10x$ (C) 6 (D) $10x - 6$ (E) $6x$

3. If the temperature in Riga rose from $-7°$C yesterday morning to $5°$C yesterday afternoon. The number of degrees the temperature rose was

(A) 2 (B) 8 (C) 10 (D) 12 (E) 14

4. A number is added to one third of itself. The result is 36. What is the number?

(A) 9 (B) 18 (C) 27 (D) 15 (E) 24

5. The average of $\frac{1}{2}$ and $\frac{2}{3}$ is

(A) $\frac{1}{5}$ (B) $\frac{1}{3}$ (C) $\frac{7}{12}$ (D) $\frac{5}{12}$ (E) $\frac{5}{6}$

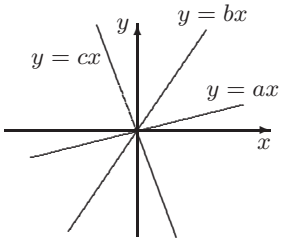
6. If two sides of a triangle are 5 cm and 7 cm, the third side cannot be

(A) 11 cm (B) 10 cm (C) 6 cm (D) 3 cm (E) 1 cm

7. When George recently visited Latvia, one Latvian *lat* was worth \$US1.50, while \$A1 was worth \$US0.60. In this case, a Latvian *lat* was worth

(A) \$A1.80 (B) \$A2.50 (C) \$A2.75 (D) \$A2.00 (E) \$A3.00

8. For the three straight lines shown, which of the following inequalities hold?



(A) $c < a < b$ (B) $b < c < a$

(C) $a < c < b$ (D) $c < b < a$

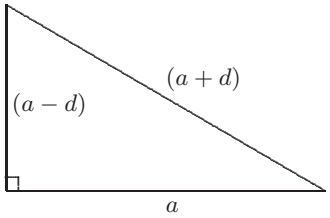
(E) $a < b < c$

S2

9. $9^{20} + 9^{20} + 9^{20}$ equals

(A) 9^{20} (B) 3^{66} (C) 9^{23} (D) 3^{41} (E) 3^{23}

10. In the diagram, find d in terms of a.



(A) $\frac{3a}{4}$ (B) $\frac{a}{2}$ (C) $\frac{a}{3}$ (D) $\frac{a}{4}$ (E) $\frac{2a}{3}$

Questions 11 - 20, 4 marks each

11. How many triangles can be drawn using the points in the diagram as vertices?

(A) 28 (B) 10 (C) 12 (D) 22 (E) 24

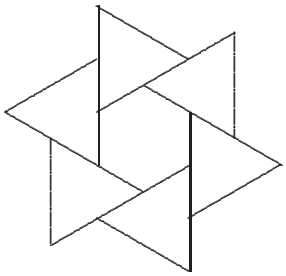
12. If f is defined by

$$f(x) = \begin{cases} x + 1, & \text{if } x < 3 \\ x + 3, & \text{if } x \geq 3 \end{cases}$$

The value of $f(4) - f(2)$ is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

13. The side of each of the equilateral triangles in the figure is twice the side of the central regular hexagon. What fraction of the total area of the six triangles is the area of the hexagon?



(A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{3}{4}$ (D) $\frac{1}{4}$ (E) $\frac{2}{3}$

S3

14. Given that $\frac{4x-y}{4x+2y} = \frac{2}{5}$, then $\frac{4x+y}{4x-2y}$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

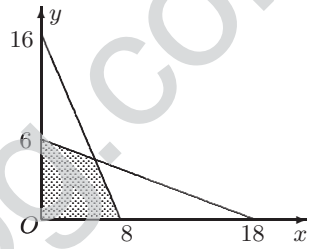
15. The smallest positive integer n for which $10^n - 1$ is a multiple of 63 is

(A) 8 (B) 7 (C) 6 (D) 5 (E) 4

16. In a mathematical competition consisting of 12 problems, 8 marks are given for each correct response, 0 marks for each incorrect response and each no response is awarded 3 marks. Vicki scored 35 marks in this competition. The largest number of incorrect responses she could have had is

(A) 1 (B) 8 (C) 11 (D) 2 (E) 7

17. In the diagram, the shaded area, in square units, is



(A) 38 (B) 24 (C) 42 (D) 20 (E) 34

18. A *good* number is the sum of a two-digit number, with distinct digits, and its reverse. For example, 110 = 37 + 73 is *good*. How many *good* numbers are perfect squares?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

19. In $\triangle PQR$, $\angle PQR - \angle PRQ = 70°$, and S and T are points on PQ and PR respectively, so that $\angle RQT = 55°$ and $\angle QRS = 40°$. The $\angle PST$ is

(A) 20° (B) 25° (C) 30° (D) 35° (E) 40°

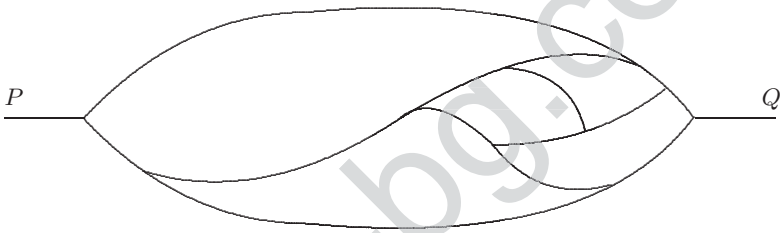
S4

20. How many numbers less than 10 000 have the product of their digits equal to 84?

(A) 24 (B) 30 (C) 42 (D) 72 (E) 84

Questions 21 - 30, 8 marks each correct response, 0 marks each incorrect response, 3 marks each no response, 30 marks minimum for this section guaranteed

21. What is the largest number of cars which can leave in some order from P and arrive in the reverse order at Q if movement is allowed only from left to right and no car can pass another as the roads are too narrow?



(A) 6 (B) 5 (C) 8 (D) 4 (E) 7

22. Let n be an odd positive integer. Then the largest positive integer k, such that

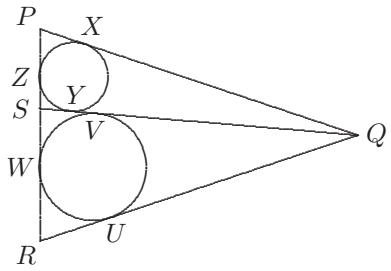
$$n^{12} - n^8 - n^4 + 1$$

is divisible by 2^k for every n is

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

S5

23. In a triangle PQR, $PQ = QR$. S is a point on PR such that $PS = 15$ and $SR = 21$. One circle touches PQ, QS and SP in X, Y and Z. Another circle touches RQ, QS and SR in U, V and W.



The length of YV is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

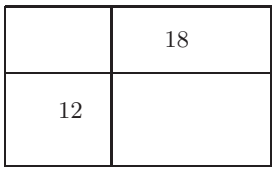
24. Given that $f(11) = 11$ and

$$f(n+3) = \frac{f(n)-1}{f(n)+1}$$

for all n, then the value of $f(2003)$ is

(A) 11 (B) $-\frac{1}{11}$ (C) 2003 (D) $-\frac{1}{2003}$ (E) $\frac{1}{121}$

25. A rectangle is divided into four smaller rectangles. The areas of two of them are shown. Of the numbers 58, 60, 63, 65, and 72, how many are possible areas of the rectangle?



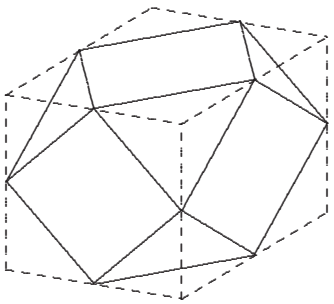
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

26. A four digit number N leaves remainder 10 when divided by 21, remainder 11 when divided by 23 and remainder 12 when divided by 25. The sum of the digits of N is

(A) 7 (B) 13 (C) 16 (D) 19 (E) 22

S6

27. All the corners are cut o a cube. These cuts are just large enough so that the new triangular faces touch at their vertices. (This figure is called a *cuboctahedron*.) All the edges are then the same length. If all the edges have length 1 cm, what is the volume of the solid?



(A) $2\sqrt{2} - \frac{2\sqrt{7}}{3}$ (B) $\frac{4\sqrt{2}-1}{3}$ (C) $\frac{4\sqrt{2}}{3}$ (D) $\frac{5\sqrt{2}}{3}$ (E) $\frac{5\sqrt{3}}{2}$

28. How many real solutions are there to the equation

$$\sqrt[3]{2x + 14} - \sqrt[3]{2x - 14} = \sqrt[3]{4}?$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

29. A 3×3 square is divided up into nine 1×1 unit squares. Di erent integers from 1 to 9 are written in these 9 unit squares. For each two squares sharing a common edge, the sum of the integers in them is calculated. The minimum possible number of di erent sums is

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

30. A set of positive integers has the properties that

- Every member in the set, apart from 1, is divisible by at least one of 2, 3 or 5.
- If the set contains $2n$, $3n$ or $5n$ for some integer n, then it contains all three and n as well.

The set contains between 300 and 400 numbers. Exactly how many does it contain?

(A) 324 (B) 343 (C) 351 (D) 360 (E) 364