AMC - 2002

SENIOR DIVISION

Questions 1 - 10, 3 marks each

1. 1.1×0.7 equals
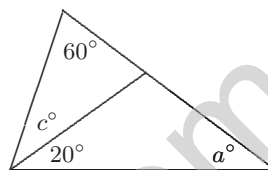
(A) 77   (B) 7.7   (C) 0.77   (D) 0.707   (E) 7.07

2. $\dfrac{3^2 + 3^2 + 3^2}{2^3 + 2^3 + 2^3}$ equals

(A) $\dfrac{3}{2}$ (B) $\dfrac{9}{8}$ (C) 1   (D) $\dfrac{3}{4}$ (E) $\dfrac{729}{512}$

3. The value of a in terms of c is

(A) $\dfrac{c}{2}$ (B) $c - 20$ (C) $c - 40$

(D) $80 - c$ (E) $100 - c$

4. The sum of $250 - 249 + 248 - 247 + 246 - \cdots + 2 - 1$ is

(A) 125   (B) 225   (C) 250   (D) 124   (E) 126

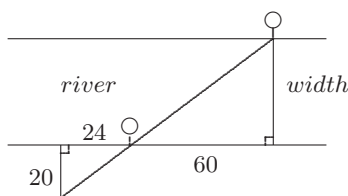**5.** If $k = 4$, $m = -0.4$ and $t = 4.4$, then the value of $\dfrac{k - m}{2t}$ is

(A) 1   (B) $\dfrac{1}{2}$ (C) $\dfrac{9}{11}$ (D) $\dfrac{9}{22}$ (E) 4.2

6. Which of the following is the largest?

(A) 2002   (B) $(20.02)^2$ (C) 200^2 (D) 2×10^4 (E) 2002×20.02

7. Two trees on either side of a river were lined up as shown in the diagram, with measurements , in metres, as shown. The width of the river, in metres, is

(A) 50   (B) 60   (C) 48

(D) 72   (E) 16

S2

8. If $a = \dfrac{3}{4}$, $b = \dfrac{4}{3}$ and $c = \dfrac{7}{9}$, which of the following is true?

(A) $b < c < a$ (B) $a < c < b$ (C) $c < a < b$

        (D) $a < b < c$ (E) $b < a < c$

9. Suppose an Australian dollar is worth 55 US cents. An Australian tourist in the USA buys an item worth \$US100 and pays \$A200. What should the change be in \$US?

(A) 5   (B) 10   (C) 15   (D) 20   (E) 25

10. The average weight of the potatoes in a bag is doubled when one extra large potato is added to the four potatoes already in the bag. The ratio of the weight of this large potato to the total weight of the other four potatoes is

(A) $3 : 2$ (B) $6 : 1$ (C) $8 : 3$ (D) $2 : 1$ (E) $8 : 1$

Questions 11 - 20, 4 marks each

11. In copying a problem involving multiplication of two numbers, James mistakenly wrote one of the numbers as 54 instead of 45. His answer was 198 larger than it should have been. The correct answer to the multiplication problem is

(A) 990   (B) 1188   (C) 405   (D) 945   (E) 1200

12. A square piece of paper 10 cm by 10 cm is folded, cut and unfolded as shown.
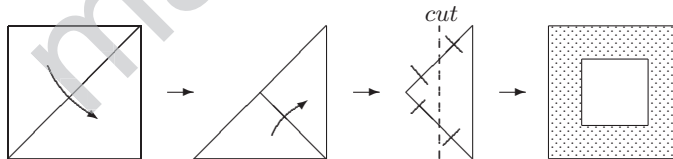
The area of the inner square (unshaded), in square centimetres, is

(A) 50   (B) 25   (C) 75   (D) 12.5   (E) 40

S3

**13.** $\sqrt{2}\sqrt{2\sqrt{2}}$ equals

(A) $3\sqrt{2}$ (B) $2\sqrt[4]{2}$ (C) 2   (D) $\sqrt[3]{2}$ (E) $\sqrt{6}$

14. On a twenty-four-hour digital clock whose display ranges from $\boxed{00:00}$ to $\boxed{23:59}$, how many times during one day does the display show a palindrome? (A palindrome is a number that is the same forwards as backwards, for example $\boxed{02:20}$, $\boxed{23:32}$.)

(A) 12   (B) 16   (C) 17   (D) 18   (E) 20

15. $UVWX$ is a parallelogram with area 24 square centimetres. M and N are the midpoints of UX and VW respectively. XNP and QMW are straight lines. The area of triangle QPO, in square centimetres, is
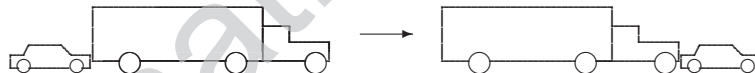
(A) 21   (B) 24   (C) 27

(D) 30   (E) 36
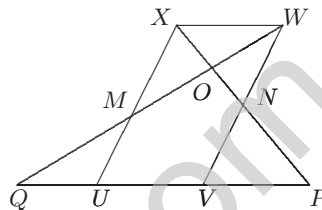
16. A class consists of 10 boys and 15 girls. Two of the boys and five of the girls are left-handed. Two students are chosen at random from the class. The probability that both are left-handed is

(A) $\dfrac{7}{15}$ (B) $\dfrac{21}{50}$ (C) $\dfrac{7}{100}$ (D) $\dfrac{18}{25}$ (E) $\dfrac{7}{25}$

17. A car 3 m long is travelling on a freeway at 110 km/h and overtakes a 17 m long truck travelling at 100 km/h.
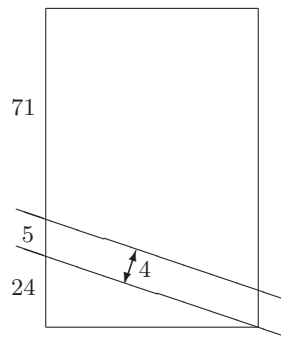
How long, in seconds, does it take the car to overtake the truck?

(A) 0.5   (B) 2   (C) 4.1   (D) 5.6   (E) 7.2

S4

18. Farmer Taylor of Burra Creek is very annoyed. A 4 m wide road has been put through one of his rectangular paddocks, splitting it into two. As a result, he has lost some of his land. If all dimensions in the figure are in metres, how much land, in square metres, has he lost?

(A) 120   (B) 150   (C) 160
      (D) 200   (E) 250

19. A sequence $a_1, a_2, a_3, \ldots$ is defined by $a_{n+2} = \dfrac{1 + a_{n+1}}{a_n}$ for $n \geq 1$.

Given that $a_1 = 2$ and $a_2 = 5$, what is the value of a_{2002}?

(A) $\dfrac{3}{5}$ (B) $\dfrac{4}{5}$ (C) 2   (D) 3   (E) 5

20. A 4×4 antimagic square is an arrangement in a square of the numbers from 1 to 16 so that the totals of each of the four rows and four columns and two diagonals are ten consecutive numbers in some order. The diagram shows an incomplete antimagic square. When it is completed, what number will replace the asterisk?

	*	14
9	3	7
12	13	5

| 10 | 11 | 6 | 4 |

(A) 1   (B) 2   (C) 8   (D) 15   (E) 16

Questions 21 - 30, 8 marks each correct response, 0 marks each incorrect response, 3 marks each no response

**21.** When we divide the numbers 272 758 and 273 437 by a two digit number N, we get remainders of 13 and 17 respectively. Find the sum of the digits of N.

(A) 6   (B) 9   (C) 10   (D) 11   (E) 12

22. There are 40 students in a class. One week a group of students from the class went on an excursion, then the next week a group of students went on a different excursion, and the next week again a group of students from the class went on another excursion. When the teacher added up the number of students that went on the first excursion, the number of students that went on the second one and the number of students that went on the third one, he got 75. The number of students that were on all three excursions is 7 and every student went on at least one excursion. What is the total number of students that have been on exactly two excursions?

(A) 14   (B) 21   (C) 26   (D) 28   (E) 33

23. The walls of a castle form a quadrilateral $PQRS$ as shown so that $PQ = 40$ m, $QR = 45$ m, $RS = 20$ m, $SP = 20$ m and $\angle PSR = 90°$. A guard must walk outside the walls so that he is always 2 m from the nearest part of the walls. The guard started walking around the castle clockwise and finally arrived at the starting point.
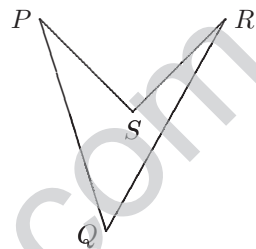
The length, in metres, of his walk was

(A) $125 + 4\pi$ (B) $121 + 5\pi$ (C) $125 + 5\pi$ (D) $121 + 6\pi$ (E) $125 + 6\pi$

24. A right angled triangle PQR has its hypotenuse PR trisected at points S and T. If $QS^2 + QT^2 = kPR^2$, then the value of k is

(A) $\dfrac{5}{9}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{1}{2}$ (D) 2 (E) $\dfrac{1}{4}$

25. An increasing sequence of integers $a_1, a_2, a_3, \ldots$ is such that

$$a_n = a_{n-1} + a_{n-2} \quad \text{for} \quad n \geq 3.$$

If $a_5 = 59$, what is the largest possible value for a_1?

(A) 4   (B) 7   (C) 10   (D) 11   (E) 12

26. Note that $1 + 2 + 3 + 45 + 6 + 78 + 9 = 144$. In how many *other* ways is it possible to make a total of 144 using only 1, 2, 3, 4, 5, 6, 7, 8, and 9 in that order and addition signs?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

S6

27. How many pairs of real numbers (x, y) satisfy the equation

$$(x + y)^2 = (x + 3)(y - 3)?$$

(A) 0   (B) 1   (C) 2   (D) 3   (E) infinitely many

28. A rectangular prism $77 \times 81 \times 100$ is cut into cubes of edge 1 by planes parallel to the faces of the prism. A given internal diagonal pierces how many of these cubes?
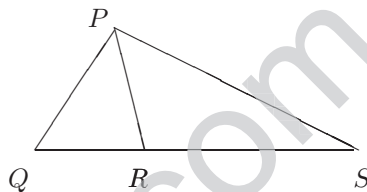
(A) 255   (B) 256   (C) 257   (D) 258   (E) 259

29. In the diagram,
$\dfrac{PQ}{PR} = \dfrac{QS}{RS}$, $\angle PQR = 38°$ and
$\angle PRQ = 46°$.
Find $\angle RPS$ in degrees.

(A) 42   (B) 43   (C) 44
       (D) 45   (E) 46

30. How many times does the digit 1 occur in the result of

$$1 + 11 + 111 + \cdots + \underbrace{111\ldots111}_{2002 \; digits}?$$

(A) 512   (B) 2002   (C) 1001   (D) 224   (E) 223