Solutions – Junior Division

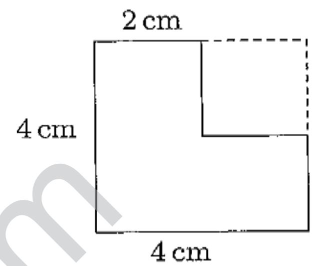
1. $2008 + 8002 = 10\,010$,

hence (E).

2. The largest is 2.2,

hence (B).

3. The perimeter of the figure is equal to the perimeter of a 4 cm by 4 cm square, and so is 16 centimetres,

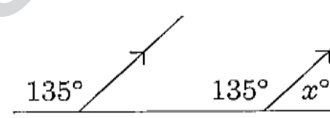


hence (D).

4. Now $199\frac{1}{2} = 200 - \frac{1}{2}$, so half that is $100 - \frac{1}{4} = 99\frac{3}{4}$,

hence (E).

5. The angle adjacent to x° is 135° (corresponding angles on parallel lines).
Since $x^\circ + 135^\circ = 180^\circ$,
$x = 45$,



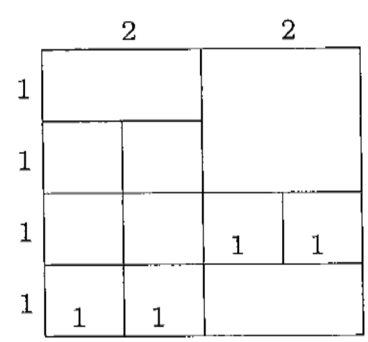
hence (E).

6. (Also I4 & S4)

$$\frac{200 \times 8}{200 \div 8} = \frac{200 \times 8 \times 8}{200} = 8 \times 8 = 64,$$

hence (D).

7. There is one 4 by 4 square.
There are no 3 by 3 squares.
There are five 2 by 2 squares.
There are eight 1 by 1 squares.
So there are $1 + 5 + 8 = 14$ squares in all,

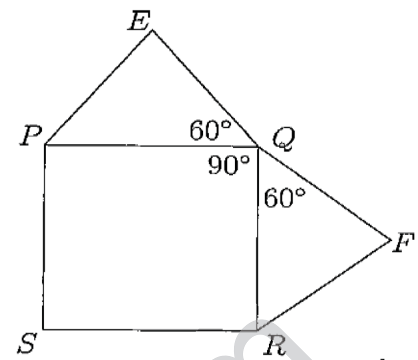


hence (D).

8. The number of minutes from 8:58 am to 9:34 am on the same day is $34 + 2 = 36$, hence (C).

9. (Also I5)
The largest such even number is 98756, so the tens digit is 5, hence (A).

10. $\angle PQE = \angle RQF = 60°$. $\angle PQR = 90°$.
So $\angle EQF = 360° - 60° - 90° - 60°$
$= 150°$,

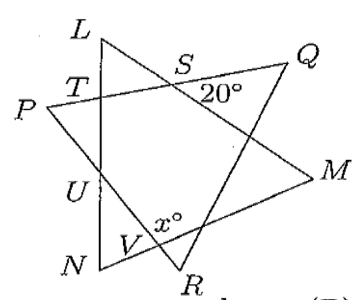


hence (D).

11. (Also I7)
Let the width be x cm. Then the length is $2x$ cm, so $x \times 2x = 72$ and $x = 6$. The perimeter is then $6 + 6 + 12 + 12 = 36$ centimetres, hence (B).

12. Since $\frac{2}{5} + \frac{1}{3} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$, then $\frac{4}{15}$ of the marbles are yellow, which is 12 marbles.
So $\frac{4}{15}$ of the total is 12, $\frac{1}{15}$ of the total is 3, and $\frac{15}{15}$, the total, is $15 \times 3 = 45$, hence (B).

13. (Also I9)
$\angle LST = \angle MSQ = 20°$, so $\angle LTS = \angle PTU = 100°$. Then $\angle PUT = \angle NUV = 20°$.
The angle $x°$ is the exterior angle of $\triangle VUN$ and so $x° = 60° + 20° = 80°$,



hence (B).

14. *Alternative 1*
The score at half-time was Newcastle 1, Melbourne 0. Three goals were scored in the second half, so the possibilities for the score at full-time are:
Newcastle 4 Melbourne 0; Newcastle 3 Melbourne 1; Newcastle 2 Melbourne 2 and Newcastle 1 Melbourne 3,
So it is not possible for Newcastle to win by 1 goal,

hence (D).

Alternative 2
A total of 4 goals were scored in the match.
So, Newcastle and Melbourne either both scored an even number of goals or both scored an odd number.
So, (D) is impossible,

hence (D).

15. Consider the different number of numbers in the sum:

- two numbers: $1+11$, $2+10$, $3+9$, $4+8$, $5+7$; 5 ways
- three numbers: $1+2+9$, $1+3+8$, $1+4+7$, $1+5+6$ $2+3+7$, $2+4+8$, $3+4+5$, 7 ways
- four numbers: $1+2+3+6$, $1+2+4+5$, 2 ways

It is not possible to add five different numbers to get 12.

This gives $5+7+2=14$ ways in all,

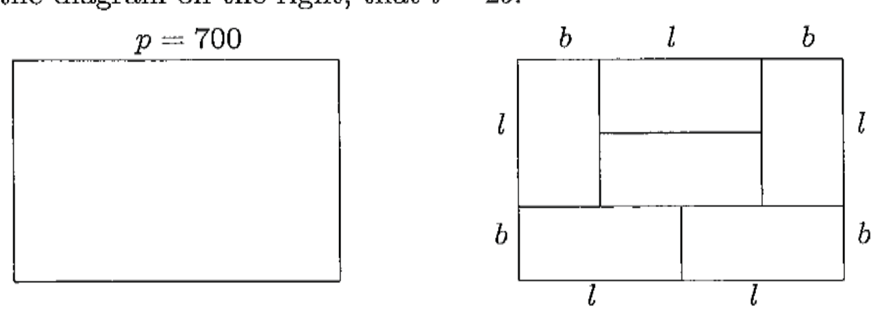
hence (C).

16. (Also I12)
The odd single-digit numbers are 1, 3, 5, 7 and 9.
The products are:
$1\times1=1$, $1\times3=3$, $1\times5=5$, $1\times7=7$, $1\times9=9$; 5 numbers
$(3\times3=9)$, $3\times5=15$, $3\times7=21$, $3\times9=27$; 3 numbers
$5\times5=25$, $5\times7=35$, $5\times9=45$; 3 numbers
$7\times7=49$, $7\times9=63$; 2 numbers
$9\times9=81$; 1 number

The total number of such numbers is $5+3+3+2+1=14$,

hence (C).

17. Let the length of each of the smaller rectangles be l and the width be b. We can see from the diagram on the right, that $l=2b$.



The perimeter of the large rectangle is $5l + 4b = 10b + 4b = 700$. So $14b = 700$ and $b = 50$. The perimeter, in metres, of each of the smaller rectangular paddocks is $2l + 2b = 200 + 100 = 300$,

hence (B).

18. There are 500 digits in the units position.
There are 491 digits in the tens position.
There are 401 digits in the hundreds position.
This gives a total of 1392 digits at 5 c each, so the cost is
$1392 \times 5 = 6960\,\text{c} = \$69.60$,

hence (D).

19. As one diagonal contains 1, 2, 3 and Y, the value of Y must be 4. Also X cannot be 3 as there is a 3 in that row, nor can it be 4 as there is already a 4 in that column.

1			
	2		
		3	X
			Y

1	4	2	3
3	2	4	1
4	1	3	2
2	3	1	4

So, the possible values of X + Y are $4 + 1 = 5$ and $4 + 2 = 6$.
We need to check that X can be 2, and this is shown to be possible in the grid on the right, so the maximum value for X + Y is 6,

hence (C).

Comment
If the diagonal restriction was removed, would the result be the same?
The only way to get a higher value is for $X, Y = 3, 4$ in some order. But, for that, $X = 4$ and $Y = 3$. The square then cannot be filled in, so the result is the same.

20. Let the first group contain x numbers.
Then the sum of the first group is $4x$.
The sum of the second group is $2x \times 10 = 20x$.
The total sum is then $24x$ and there are $3x$ numbers , so the average is $\frac{24x}{3x} = 8$,

hence (D).

21. Since 2 metres = 200 centimetres, there are $\frac{200}{5} = 40$ of the 5-centimetre cubes along each edge of the 2-metre cube.

So there are $40 \times 40 \times 40$ of the 5-centimetre cubes.

So, when they are stacked one on top of the other,

$$\text{height} = 40 \times 40 \times 40 \times 5\,\text{cm} = 320\,000\,\text{cm} = 3200\,\text{m} = 3.2\ \text{km},$$

hence (D).

22. (Also I18)
Let the number be x. The given conditions tell us that $x+1$ is divisible by 2, 3 and 5. It is therefore divisible by 30.
The largest multiple of 30 which is ≤ 2008 is 1980, so 1979 is the number, and the sum of the digits is 26,

hence (A).

23. (Also I21 & S18)
The amount of water collected is proportional to the areas of the two roofs. So

volume collected on farmhouse roof : volume collected on barn roof

is $200 : 80 = 5 : 2$.

So, if Farmer Taylor is to collect as much water as possible, the empty space in the tanks has to be in the same ratio $5 : 2$.

Currently, there is $100 - 35 = 65\,\text{kL}$ available in the farmhouse tank and $25 - 13 = 12\,\text{kL}$ in the barn tank.

Now, the ratio $65 : 12$ is greater than $5 : 2$, so we must pump some water from the barn tank into the house tank.

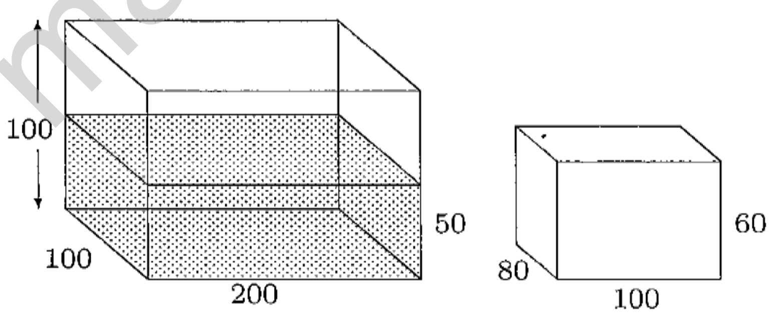
If we pump $x\,\text{kL}$ from the barn tank into the house tank, then the empty space in the house tank is $65 - x\,\text{kL}$ and the empty space in the barn tank is $12 + x\,\text{kL}$.

So, we want $65 - x : 12 + x = 5 : 2$, which gives $\dfrac{65-x}{5} = \dfrac{12+x}{2}$,
$130 - 2x = 60 + 5x$, $7x = 70$ and $x = 10$.

So, to collect the maximum amount of water possible we must pump 10 kL from the barn tank into the farmhouse tank,

hence (D).

24. (Also I15 & S10)
The volume of the water is $100 \times 200 \times 50 = 1\,000\,000\ \text{cm}^3$.



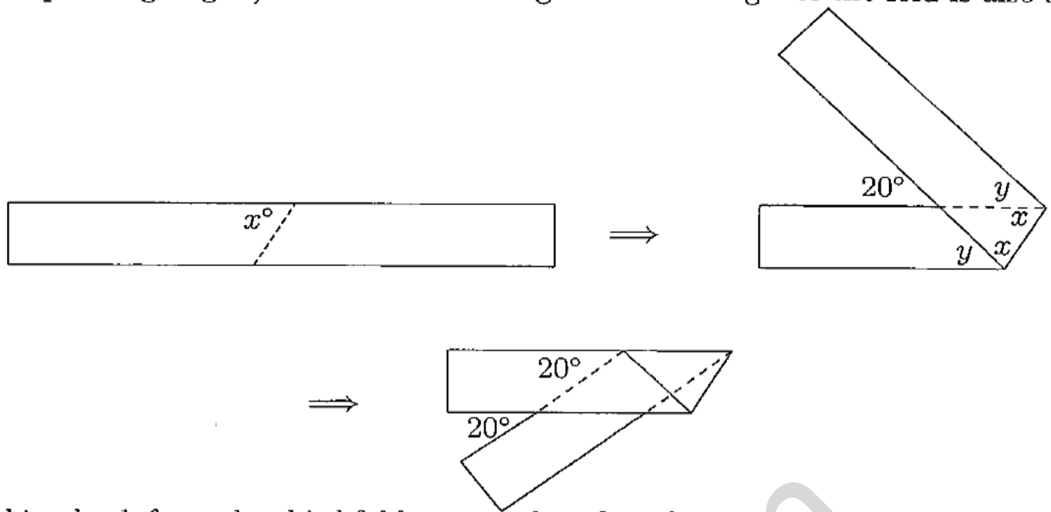
The volume of the prism is $80 \times 100 \times 60 = 480\,000\,\text{cm}^3$.
So, when the prism is placed in the tank, the new height of water is

$$\frac{1480000}{20000} = 74\ \text{cm}.$$

The prism is 60 cm high so is covered by 14 centimetres of water,

hence (B).

25. Looking at the second fold, we can see that the two angles marked $y°$ are equal (corresponding angles). So the bottom angle in the triangle of the fold is also $x°$.



Working back from the third fold, we see that the other angle in the triangle is 20°, so the triangle in the fold is isosceles with angles of $x°$, $x°$ and 20°, so $2x + 20 = 180$ and $x = 80$,

hence (E).

26. *Alternative 1*

The numbers can only contain the digits 1, 3, 7 and 9, as otherwise one or more would be composite.

Let the two numbers be $10a + b$ and $10b + a$.
Their sum is then $10a + b + 10b + a = 11(a + b)$.

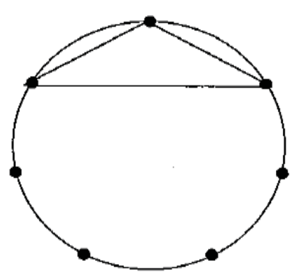
The case $a = b = 9$ is ruled out as 99 is not prime.
The next largest value of $a + b$ is $9 + 7 = 16$, corresponding to the numbers 97 and 79, both of which are prime.

Hence the maximum sum is $97 + 79$ $(= 11 \times 16) = 176$.

Alternative 2

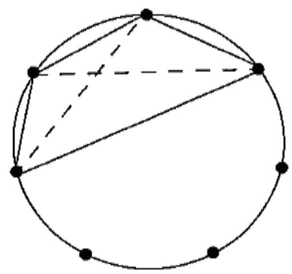
99 is not prime, nor is 98 but 97 is. Also, 79 is prime, so the largest sum is $97 + 79 = 176$.

27. Consider a diagonal that subtends two sides of the heptagon.



There is only one obtuse-angled triangle with this diagonal as its longest side. Since there are 7 such diagonals, there are 7 obtuse-angled triangles with their longest sides subtending two sides of the heptagon.

Now consider a diagonal that subtends three sides of the heptagon.

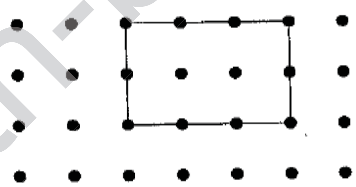


There are two-obtuse angled triangles with this diagonal as their longest side. Since there are again 7 such diagonals, there are $7 \times 2 = 14$ obtuse-angled triangles with their longest sides subtending three sides of the heptagon.

There are no more obtuse-angled triangles.

Clearly this way we have counted every obtuse-angled triangle once and only once. So the answer is 21.

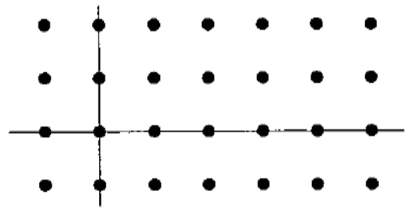
28. (Also I27)
Alternative 1
Consider the grid points (vertices of the unit cubes) on one layer of the figure. There are $7 \times 4 = 28$ such points.



Consider the different rectangles (including squares) which can be made with these grid points.
On the longer side there are 7 points from which we can choose two, as we can choose the first with six others to give 6, the second with five others to give 5, the third with four others to give 4, the fourth with three others to give 3, the fifth with two others to give 2 and the sixth with one other to give $6 + 5 + 4 + 3 + 2 + 1 = 21$ ways of choosing the two vertices of the rectangle.
The other two points on the shorter side can be chosen in the same fashion, the first with three others, the second with two others and the third with one, giving $3 + 2 + 1 = 6$ ways.
Each of these ways can be combined with each of the ways in the longer side, so there are $21 \times 6 = 126$ ways of choosing a rectangle in one layer of the grid points.
Now, the top and bottom vertices of a prism can be chosen in a similar way from the four layers of grid points, the first with three others giving 3, the second with two others giving 2 and the third with one other giving 1, a total of $3 + 2 + 1 = 6$ ways.

Each of these can be combined with each way of selecting a rectangle in a layer, so there are $126 \times 6 = 756$ possible rectangular prisms in the grid.

Alternative 2
There are $7 \times 4 \times 4$ grid points in the figure.
A prism is defined by 2 points (opposite ends of a diagonal).
The first point can be chosen in $7 \times 4 \times 4$ ways and the second in $6 \times 3 \times 3$ ways, (no two can be the same and we must exclude points in the same horizontal and vertical plane as the chosen point).



So there are $7 \times 4 \times 4 \times 6 \times 3 \times 3$ ways to choose the points, but we have chosen the same points 8 times, (there are 4 diagonals of each prism and the end points can be interchanged as well).
So there are $\frac{7 \times 4 \times 4 \times 6 \times 3 \times 3}{8} = 756$ such different rectangular prisms in the figure.

29. (Also I29 & S27)
We need to find the biggest sums we can get for a given number of terms. Some experiments:

1 term	$1 = 1$
2 terms	$2 = 1 + 1$
3 terms	$4 = 1 + 2 + 1$
4 terms	$6 = 1 + 2 + 2 + 1$
5 terms	$9 = 1 + 2 + 3 + 2 + 1$
6 terms	$12 = 1 + 2 + 3 + 3 + 2 + 1$

We can see that we get different formulas for odd or even numbers of terms.

For $2n$ terms, the maximum sum is $n(n+1)$.
For $2n+1$ terms , the maximum sum is $(n+1)^2$.

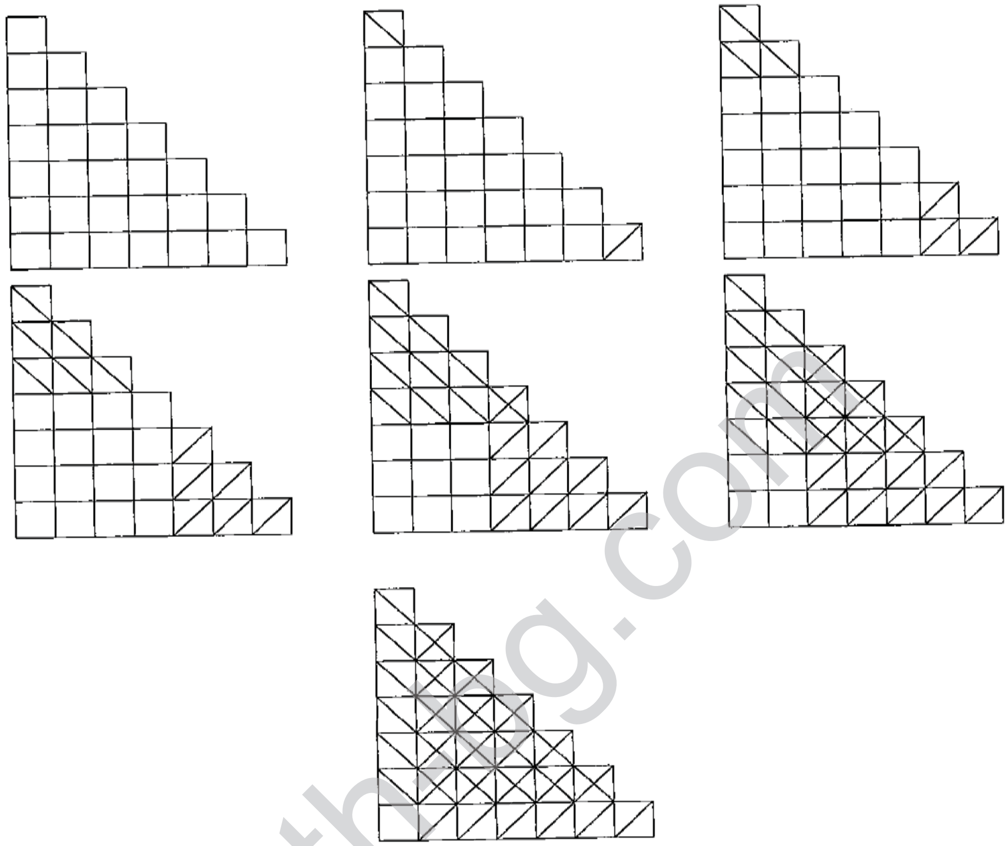
Now $44 \times 45 = 1980$ and $45 \times 45 = 2025$, so 88 terms will not get us there, but 89 looks as though it should.

We can show it does, by starting with the biggest 88 term sum:

$$1980 = 1 + 2 + 3 + \ldots + 88 + 88 + 87 + \ldots + 3 + 2 + 1$$

This is 28 short of what we want, so put in a term of 28 next to one of the two 28s already there and we have a sum of 2008 with a minimum of 89 terms.

30. *Alternative 1*
Here the 7 layers are shown, front to back, all bricks consistent with the front view. Then cross out the bricks  not consistent with the above view, and the bricks  not consistent with the side view.



The bricks left constitute the bricks in the monument, so there are
$28 + 26 + 22 + 16 + 9 + 4 + 1 = 106$ bricks in the monument.

Alternative 2
If we consider the number of bricks in each position consistent with all views, the number of bricks in each position shown on the aerial view is

1						
2	2					
3	3	3				
4	4	4	4			
5	5	5	4	3		
6	6	5	4	3	2	
7	6	5	4	3	2	1

This gives $2 \times 1 + 4 \times 2 + 6 \times 3 + 7 \times 4 + 5 \times 5 + 3 \times 6 + 7 = 106$ bricks in the monument.