Solutions – Junior Division

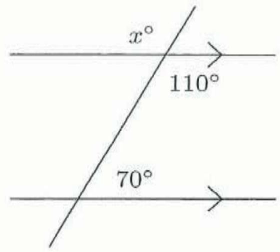
1. $95 - 83 = 12$,

hence (D).

2. $0.5 = \frac{5}{10} = \frac{1}{2}$,

hence (E).

3. From the diagram we can see that the angle of 110° is cointerior



with the angle of 70° and then this angle is vertically opposite to the angle $x°$, so $x = 110$,

hence (E).

4. (Also S1)

$$\frac{6 \times 25}{3 \times 5 \times 2} = \frac{6 \times 25}{6 \times 5} = \frac{25}{5} = 5,$$

hence (D).

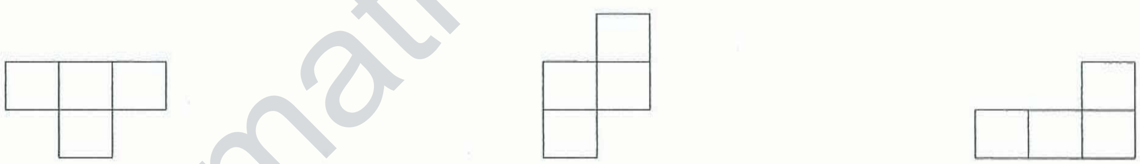
5. The difference is $901 - 789 = 112$ km,

hence (D).

6. The change after buying 7 bottles at 70 c is
$\$50 - 7 \times .70 = 50 - 4.90 = \$45.10$,

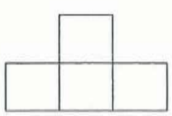
hence (A).

7. Considering these three different tetrominoes:-

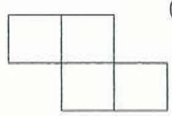


We can see that (A) is the first one rotated through 180°,

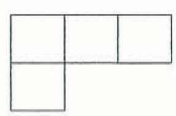
(A)



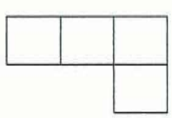
(B)



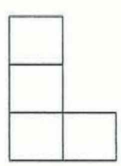
(C)



(D)



(E)



(B) is the second one rotated 90° clockwise or anticlockwise, (C) is the third rotated 180°, (E) is the third rotated 90° clockwise, but (D) cannot be obtained from the third by a rotation in the plane,

hence (D).

8. (Also I5)
Since $64 = 4 \times 4 \times 4$, the length of the edge of a face is 4 cm, and the area of a face in square centimetres is $4 \times 4 = 16$,

hence (B).

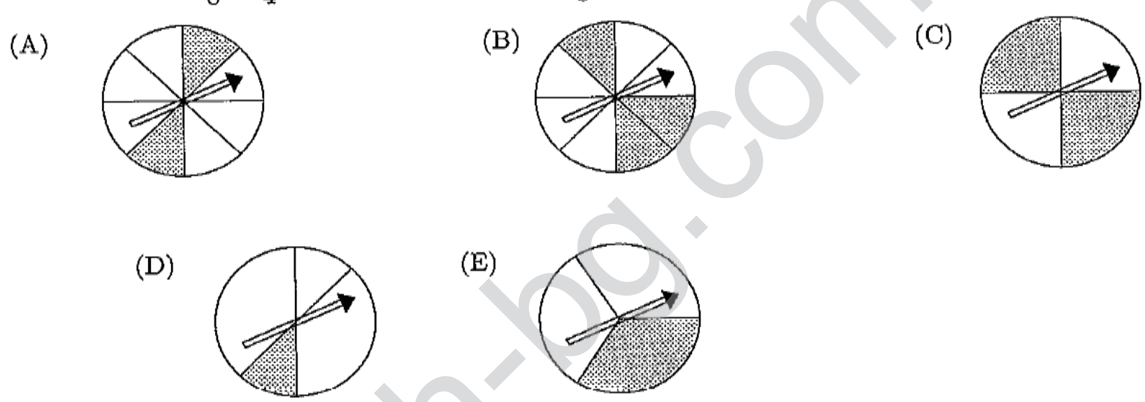
9. $\frac{3}{5} - \frac{2}{10} + \frac{3}{15} - \frac{4}{10} = \frac{3}{5} - \frac{1}{5} + \frac{1}{5} - \frac{2}{5} = \frac{1}{5}$,

hence (C).

10. (Also I6)
The 5 numbers are averaged, so their sum is $5 \times 4 = 20$. Since $1 + 2 + 3 + 4 = 10$ it follows that the fifth number is 10,

hence (E).

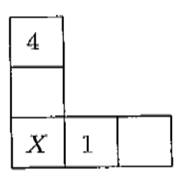
11. (Also I4)
Spinner (A) has $\frac{2}{8} = \frac{1}{4}$ shaded, spinner (B) has $\frac{3}{8}$ shaded,



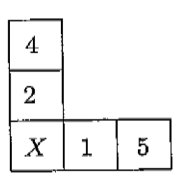
spinner (C) has $\frac{2}{4} = \frac{1}{2}$ shaded, spinner (D) has $\frac{1}{8}$ shaded and spinner (E) has $\frac{1}{3}$ shaded, so the spinner with a 1 in 4 chance is (A),

hence (A).

12. If we place a number in the position marked X, for the row and column sum to be the same, the number in the vertical column must be 3 less than the one in the horizontal row.



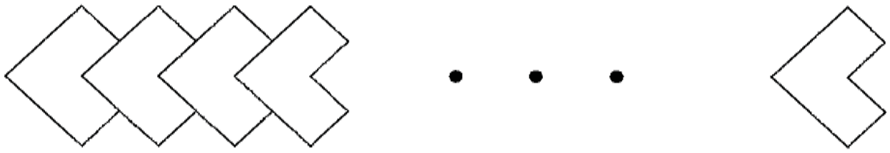
The only two numbers left which differ by 3 are 5 and 2, so the 2 must go underneath the 4, the 5 to the right of the 1 as $4 + 2 = 5 + 1$. Thus the 3 must be in the bottom corner.



The sum of the row or column is $3 + 2 + 4 = 3 + 1 + 5 = 9$,

hence (B).

13. If the fifty tiles are arranged as shown, the 48 tiles other than the end tiles each contribute 4 cm to the perimeter, and the two end tiles each contribute 6 cm to the perimeter.



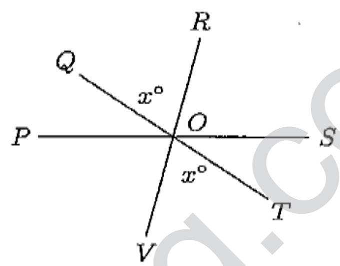
The perimeter, in centimetres, is $48 \times 4 + 2 \times 6 = 192 + 12 = 204$,

hence (B).

14. Using a 20 c coin we can get 35 c by {20, 10, 5}; {20, 5, 5, 5} that is 2 ways.
Using 10 c and 5 c coins, we get 35 c by {10, 10, 10, 5}; {10, 10, 5, 5, 5}; {10, 5, 5, 5, 5, 5} that is 3 ways.
Using 5 c coins only, we get 35 c in just one way, {5, 5, 5, 5, 5, 5, 5}.
Thus the total number of ways is $2 + 3 + 1 = 6$,

hence (B).

15. (Also I8 & S3)
In the diagram, $\angle POR = 120°$ and $\angle QOS = 145°$.



Let $\angle TOV = \angle QOR = x°$.
Then $\angle POR + \angle QOS = \angle POS + x° = 180° + x°$, (since POS is a straight line).
So $120 + 145 = 180 + x$ and $x = 85$,

hence (C).

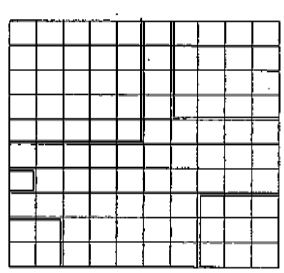
16. (Also I11)
There are 365 days in the year 2006.
Thus the middle day of the year is the 183rd day of the year.
January has 31 days, February 28, March 31, April 30, May 31 and June 30, giving a total of 181 days in the first 6 months, so the middle date is 2 days on, on 2nd July,

hence (D).

17. $770 = 10 \times 77 = 2 \times 5 \times 7 \times 11$.
The only possible combination of factors to give a number from 13 to 19 is $2 \times 7 = 14$, so the teenager is 14,

hence (B).

18. We cannot cut out a $6 \times 6 \times 6$ cube and a $5 \times 5 \times 5$ cube at the same time, as there would have to be overlap on at least one layer, as $5 + 6 > 10$, the number of layers in the large cube.



The end view shows how cubes $1 \times 1 \times 1$, $2 \times 2 \times 2$, $3 \times 3 \times 3$, $4 \times 4 \times 4$ and $5 \times 5 \times 5$ can be cut from the larger cube, so the largest possible is $5 \times 5 \times 5$,

hence (E).

19. The smallest number is 3456, and there are 6 numbers starting with 3, 6 numbers starting with 4, 6 numbers starting with 5 and 6 with 6.
The thirteenth number, when arranged in ascending order, is thus the first one starting with 5 which is 5346,

hence (C).

20. We can see that, as the other angles in the folded triangle are 45°, the second fold produces a right-angled triangle with sides of 2 cm about the right angle.

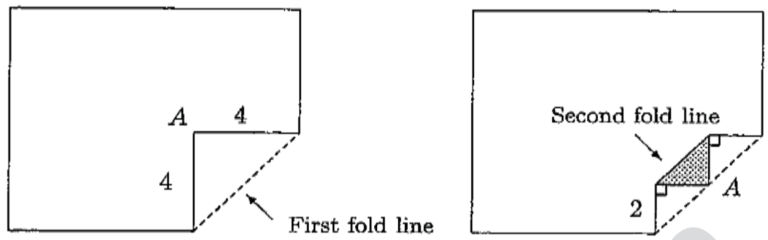


figure 1 *figure* 2

The area of the shaded triangle, in square centimetres, is then $\frac{1}{2} \times 2 \times 2 = 2$,

hence (B).

21. Consider the 6 by 6 block as shown. All the outside apartments have exterior views, so we need only deal with the 16 interior squares.

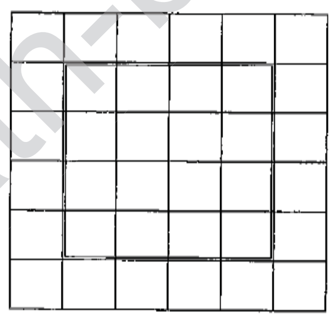
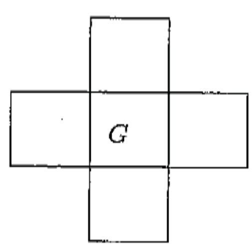


figure1

Putting in one garden will remove one flat and bring light to (at most) four other flats.



This complying arrangement will deal with at most five of the internal squares and so, as $3 \times 5 < 16$, at least four gardens are required.

It is relatively easy to find such a solution with 4 gardens, as shown in figure 2.

		G			
				G	
	G				
			G		

figure2

So the smallest number of gardens required is 4, hence (A).

22. The first few powers of 2 are:- $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$ and $2^6 = 64$.
So we can see that, for the last digit, there is a cycle of length 4. So the last 2^{2006} is the second in a cycle of 4 and so ends with 4, hence (C).

23. (Also I22 & S20)
(Alternative 1)
For each of the three choices to place R in row one, there are two choices to place R in row two and then only one choice in row 3.
So, there are 6 ways of placing the Rs.

R		
		R
	R	

For each of these, there are two choices to place W in row one, then one in row two and one in row three, and for each of these, the placement of B is determined.

Hence the number of different patterns is $6 \times 2 \times 1 = 12$, hence (D).

(Alternative 2)
There are 6 ways of placing 3 colours in the top row.
Then there are 2 choices for colours in the second row and the final row is then determined, so there are $6 \times 2 = 12$ ways, hence (D).

24. (Also I17 & S15)
If a 3-digit number has digits all the same then the number is one of the numbers 111, 222, 333, ... , 999.

Each of these numbers is divisible by 111 and the prime factors of 111 are 3 and 37.

So, one of the two-digit numbers must be a multiple of 3 and the other must be a multiple of 37.

We cannot obtain the products 111, 222 or 333 multiplying two two-digit numbers. We can get

$(1 \times 37) \times (4 \times 3) = 444$,
$(1 \times 37) \times (5 \times 3) = 555$,
$(1 \times 37) \times (6 \times 3) = 666$,
$(1 \times 37) \times (7 \times 3) = 777$,
$(1 \times 37) \times (8 \times 3) = 888$,
$(1 \times 37) \times (9 \times 3) = 999$,
$(2 \times 37) \times (4 \times 3) = 888$,

with 888 being the only one obtainable in two ways, so the number of pairs is 7,

hence (C).

25. If the leftmost digit of N is a and $2 \leq a \leq 7$, the leftmost digit of $6N$ will be one of the digits: 1, 2, 3, 4.
Now assume that the leftmost digit of N is 8. Then $8 \times 10^k \leq N < 9 \times 10^k$ for some non-negative integer k. Hence $48 \times 10^k \leq 6N < 54 \times 10^k$, and therefore either the leftmost digit of $6N$ or its second leftmost digit is one of the digits: 0, 1, 2, 3, 4.
Therefore the leftmost digit of N is not 8, so the leftmost digit is either 1 or 9.
So the digit in question must be 9.

hence (E).

26. In the units position, there will be 9 lots of $0 + 1 + 2 + 3 + \cdots + 9$.
In the tens position, there will be 10 lots of $1 + 2 + 3 + \ldots + 9$.
The sum of the digits is then
$9(0 + 1 + 2 + 3 + \cdots + 9) + 10(1 + 2 + 3 + \cdots + 9) = 9 \times 45 + 10 \times 45 = 855$.

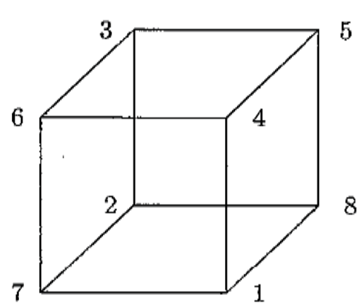
27. (Also I27 & S26)
There are 18 possible digit sums for two-digit numbers: 1, 2, 3, ..., 17, 18.
There is only one two-digit number with digit sum 1, namely 10; and there is only one two-digit number with digit sum 18, that is 99.
Therefore the largest number of two-digit numbers that can be written on the whiteboard, such that no three numbers have the same digit sum, is $1 \times 2 + 2 \times 16 = 34$.
Thus the smallest number of students in the class for the teacher to be correct is 35.

28. Consider the $5 \times 5 \times 4$ block to be 4 layers of 5×5 cubes.
Consider the first layer. If we start with a red cube in a corner we must alternate red with white and will get 13 red cubes and 12 white.
The next layer must have white in the corners and so will have 13 white and 12 red.
The remaining two layers are the same as the first two, so there are equal numbers of red and white blocks, and from the symmetry, there is an equal number of red and white faces on the outside. Thus there is also an equal number of red and white faces in the interior.

The total number of faces is $6 \times 5 \times 5 \times 4 = 600$.
The number of faces showing on the outside is
$2 \times 5 \times 5 + 4 \times 4 \times 5 = 50 + 80 = 130$.

So there are $600 - 130 = 470$ faces in the interior, half of which are red, hence there are $470 \div 2 = 235$ red faces in the interior of the block.

29. (Also I25 & S22)
There are 6 faces and hence 6 face sums. Since each vertex lies on 3 faces, the sum of all the face sums of the cube is $3(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = 108$, so if there are 6 equal face sums, that sum must be $108 \div 6 = 18$.



The figure gives an example of 6 equal face sums, which is the maximum number possible.

30. $8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 = 2^7 \times 3^3 \times 5 \times 7 \times 11 \times 13$. (1)
So the product we are looking for contains multiples of 11 and 13 but no other primes greater than 13.

Consider the multiples

11: 11, 22, 33, 44, 55, 66, 77, 88, 99, ...
13: 26, 39, 52, 65, 78, 91, ...

We are looking for a multiple of each which does not contain a prime or multiple of a prime > 13 between those two numbers, for example, 22 and 26 will not work as there is the prime 23 between them.
Of the pairs above, 65 and 66 are adjacent and so we consider a product containing 65×66.
From (1), this leaves the factors $2^6 \times 3^2 \times 7$.

Now $2^6 \times 3^2 \times 7 = 64 \times 63$ and so the product we are looking for is $63 \times 64 \times 65 \times 66$ and the smallest of these factors is 63. (It is not hard to show there is no other such factor equation).