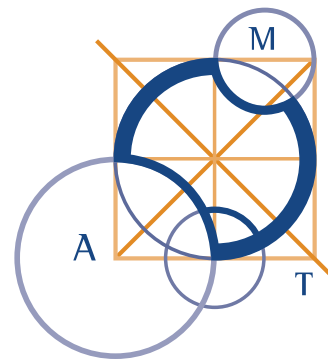


AUSTRALIAN MATHEMATICS COMPETITION

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST



THURSDAY 6 AUGUST 2009

INTERMEDIATE DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 9 AND 10

TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the **Answer Sheet** carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the Answer Sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMC reserves the right to re-examine students before deciding whether to grant official status to their score.

Intermediate Division

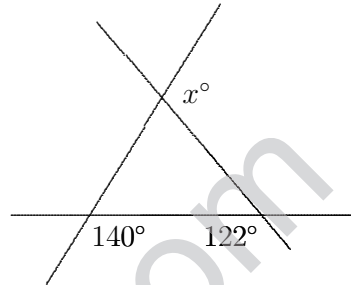
Questions 1 to 10, 3 marks each

1. $(2000 + 9) + (2000 - 9)$ equals

(A) 4000 (B) 4009 (C) 200 (D) 2000 (E) 5000

2. In the diagram, x equals

(A) 140 (B) 122 (C) 80
(D) 90 (E) 98



3. The value of $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$ is

(A) $\frac{1}{5}$ (B) $\frac{5}{7}$ (C) $\frac{1}{6}$ (D) $\frac{1}{15}$ (E) $\frac{1}{60}$

4. Which of the following has the largest value?

(A) $\frac{1}{3}$ (B) $\frac{1}{3} + \frac{1}{3}$ (C) $\frac{1}{3} \times \frac{1}{3}$ (D) $\frac{1}{3} - \frac{1}{3}$ (E) $\frac{1}{3} \div \frac{1}{3}$

5. Which of the following values can replace the box so that

$$0.1 \times 0.2 \times 0.3 \times 0.4 \times \boxed{} = 0.12?$$

(A) 500 (B) 50 (C) 5 (D) 0.5 (E) 0.05

6. If $3^k = 9^{30}$ then k equals

(A) 15 (B) 30 (C) 40 (D) 60 (E) 90

7. $(x - y) - 2(y - z) + 3(z - x)$ equals

- (A) $-2x - 3y + 5z$ (B) $-2x - 3y - z$ (C) $4x + y - z$
(D) $4x + 3y - z$ (E) $2x + 3y - 5z$

8. The point $(k, 17)$ lies on the line joining the points $(1, 5)$ and $(4, 11)$. The value of k is

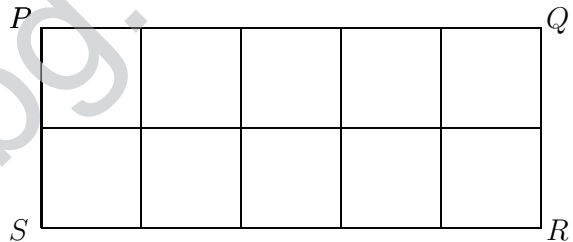
- (A) 37 (B) 14 (C) 8 (D) 6 (E) 7

9. Paperback books cost \$5 each and hardcover books cost \$7 each. I spend exactly \$86 on books of these two types. What is the maximum number of books that I could have bought?

- (A) 10 (B) 14 (C) 16 (D) 18 (E) 20

10. The figure $PQRS$ is a rectangle divided into 10 squares as shown. The perimeter of this rectangle is 21 centimetres. In centimetres, what is the perimeter of each square?

- (A) 2.1 (B) 3 (C) 6
(D) 8.4 (E) 12



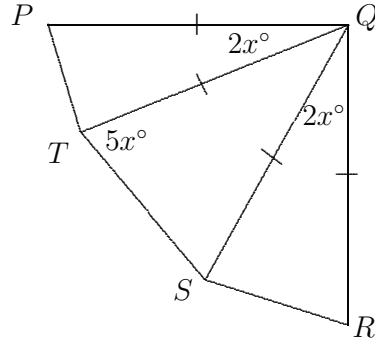
Questions 11 to 20, 4 marks each

11. In a school of 1000 students, 570 are girls. One-quarter of the students travel to school by bus and 313 boys do **not** go by bus. How many girls travel to school by bus?

- (A) 7 (B) 63 (C) 153 (D) 180 (E) 133

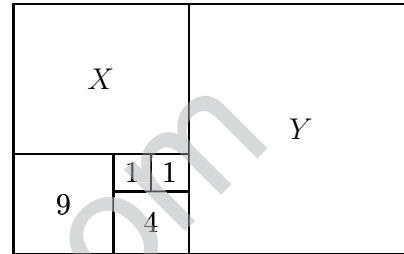
12. In the diagram, triangles PQT , QTS and QRS are isosceles and $\angle PQR$ is a right angle. Angles PQT and RQS are $2x^\circ$ and angle QTS is $5x^\circ$. The value of x is

(A) 10 (B) 12 (C) 14
(D) 15 (E) 20



13. This pattern consists of squares. Areas, in square units, of four of the squares are shown. Given that X and Y are also squares, their areas are respectively

(A) 16, 25 (B) 16, 36 (C) 25, 36
(D) 25, 64 (E) 25, 100



14. Two numbers P and Q are such that P is 40% greater than Q . The ratio $P : Q$ is

(A) 40 : 1 (B) 5 : 7 (C) 5 : 3 (D) 5 : 2 (E) 7 : 5

15. What is the last digit of 6×8^{2009} ?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

16. Two dice are each numbered from 1 to 6, but are biased so that each is twice as likely to land on any of the even numbers as on any of the odd numbers. The two dice are rolled and the numbers multiplied together. What is the probability that the resulting product is odd?

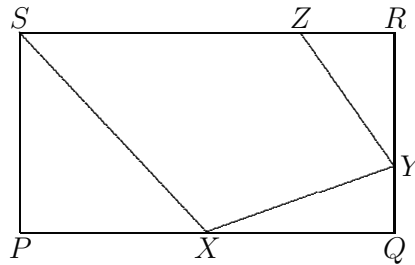
(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{1}{3}$ (D) $\frac{4}{9}$ (E) $\frac{2}{3}$

17. An *eyebrow* is an arrangement of the numbers 1, 2, 3, 4 and 5 such that the second and fourth numbers are each bigger than both their immediate neighbours. For example, (1, 3, 2, 5, 4) is an eyebrow and (1, 3, 4, 5, 2) is not. The number of eyebrows is

(A) 16 (B) 12 (C) 15 (D) 24 (E) 18

18. $PQRS$ is a rectangle. X is halfway along PQ , Y is a third of the way along QR and Z is a quarter of the way along RS . What fraction of the area of $PQRS$ is represented by the quadrilateral $XYZS$?

- (A) $\frac{1}{2}$ (B) $\frac{7}{12}$ (C) $\frac{2}{3}$
(D) $\frac{3}{4}$ (E) $\frac{3}{5}$



19. The difference between a positive fraction and its reciprocal is $\frac{9}{20}$. The sum of the fraction and its reciprocal is

- (A) $\frac{41}{40}$ (B) $\frac{20}{9}$ (C) $\frac{25}{16}$ (D) $\frac{41}{20}$ (E) 5

20. A circular bottle with dimensions, in centimetres, is shown partially filled with water (figure 1).

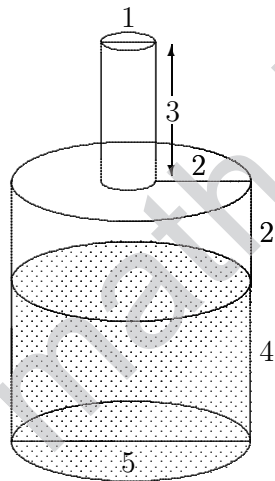


figure 1

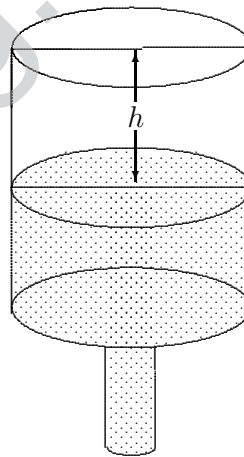


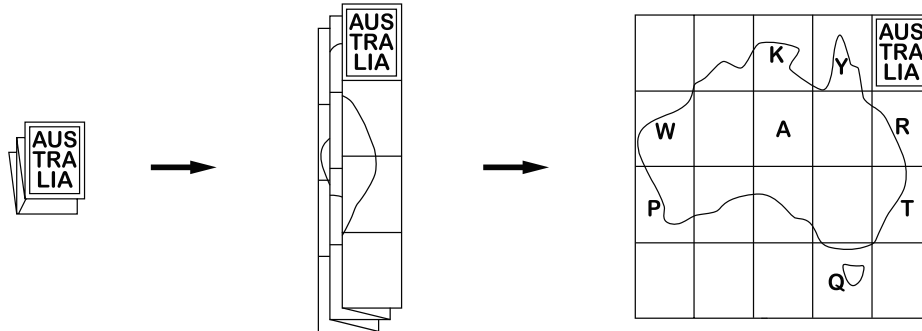
figure 2

The bottle is sealed and then turned upside down (figure 2). The height h , in centimetres, of the air in the upturned bottle is

- (A) 2 (B) $2\frac{1}{3}$ (C) $2\frac{7}{25}$ (D) $2\frac{1}{5}$ (E) $2\frac{3}{25}$

Questions 21 to 25, 5 marks each

21. I bought a map of Australia, unfolded it and marked eight places I wanted to visit.



I then refolded the map and placed it back on the table as it was. In what order are my marks stacked from top to bottom?

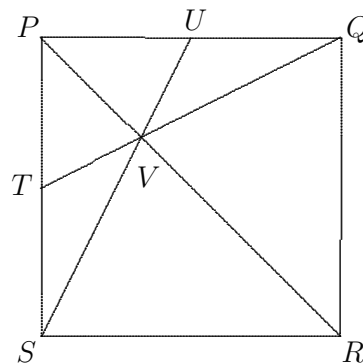
- (A) RTYQKAWP (B) YKRAWTPQ (C) RTQYKAWP
(D) YKTPRAWQ (E) YKWARTPQ

22. A palindromic number is a 'symmetrical' number which reads the same forwards as backwards. For example, 55, 101 and 8668 are palindromic numbers. There are 90 four-digit palindromic numbers. How many of these four-digit palindromic numbers are divisible by 7?

- (A) 7 (B) 9 (C) 14 (D) 18 (E) 21

23. $PQRS$ is a square. T and U are mid-points of the sides PS and PQ respectively. TQ and SU intersect at V . What fraction of the area of the square is the area of quadrilateral $QVSR$?

- (A) $\frac{1}{2}$ (B) $\frac{5}{8}$ (C) $\frac{2}{3}$
(D) $\frac{3}{4}$ (E) $\frac{5}{9}$



24. In 3009, King Warren of Australia suspects the Earls of Akaroa, Bairnsdale, Claremont, Darlinghurst, Erina and Frankston are plotting a conspiracy against him. He questions each in private and they tell him:

Akaroa: Frankston is loyal but Erina is a traitor.

Bairnsdale: Akaroa is loyal.

Claremont: Frankston is loyal but Bairnsdale is a traitor.

Darlinghurst: Claremont is loyal but Bairnsdale is a traitor.

Erina: Darlinghurst is a traitor.

Frankston: Akaroa is loyal.

Each traitor knows who the other traitors are, but will always give false information, accusing loyalists of being traitors and vice versa. Each loyalist tells the truth as he knows it, so his information on traitors can be trusted, but he may be wrong about those he claims to be loyal.

How many traitors are there?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

25. Which of the following cannot be the last digit of the sum of the squares of seven consecutive numbers?

- (A) 3 (B) 5 (C) 6 (D) 7 (E) 8

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

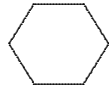
Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. What is the smallest positive integer which, when divided by each of 2, 3, 4, 5, 6 and 7, will give in each case a remainder that is one less than the divisor?

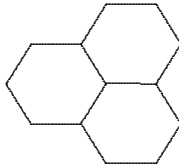
27. We say a number is *ascending* if its digits are strictly increasing. For example, 189 and 3468 are ascending while 142 and 466 are not. For which ascending 3-digit number n (between 100 and 999) is $6n$ also ascending?

28. A magician deposits the same number of rabbits (at least one) at each of five houses. To get to the first house he crosses a magic river once, and to get to any house from another, he also crosses a magic river once. Each time he crosses a magic river, the number of rabbits he has doubles. He has no rabbits left when he leaves the fifth house. What is the minimum number of rabbits he could have at the start?

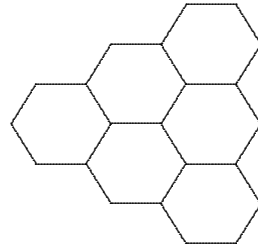
29. Consider this sequence of patterns made from hexagons.



pattern 1



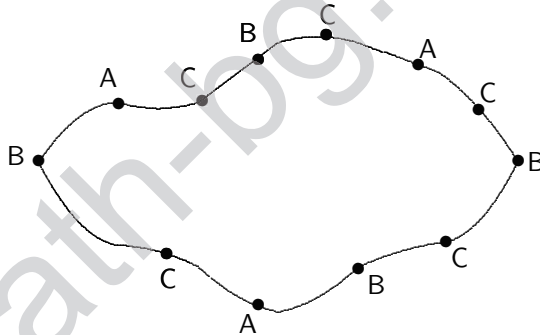
pattern 2



pattern 3

The first pattern consists of 6 line segments, the second of 15 and the third of 27. How many line segments are in the 11th pattern?

30. The country of Big Wally has a railway which runs in a loop 1080 km long. Three companies, A, B and C run trains on the track and plan to build stations. Company A will build three stations, equally spaced at 360 km intervals. Company B will build four stations at 270 km intervals and Company C will build five stations at 216 km intervals.



The government tells them to space their stations so that the longest distance between consecutive stations is as small as possible. What is this distance in kilometres?

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