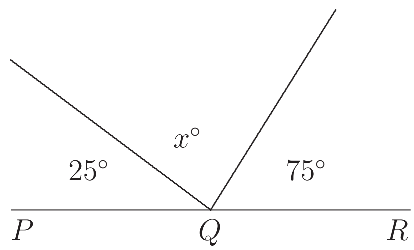
**Intermediate Division**

**1.** $\dfrac{8 \times 9}{3} = 8 \times 3 = 24$,

hence (C).

**2.** (Also J3)
$PQR$ is a straight line, so $x + 25 + 75 = 180$.




So $x = 180 - 100 = 80$.

hence (C).

**3.**

$$\begin{aligned} 110 + x &= 97 + y \\ 110 - 97 + x &= y \\ 13 + x &= y, \end{aligned}$$

hence (A).

**4.** Two chocolates at \$1.35 each cost \$2.70, so Andy should get \$2.30 change from \$5,
hence (E).

**5.** (Also J10 & S4)
The fractions $\frac{7}{15}$, $\frac{3}{7}$ and $\frac{4}{9}$ are each less than $\frac{1}{2}$. The fraction $\frac{6}{11} > \frac{1}{2}$ and is then the largest,

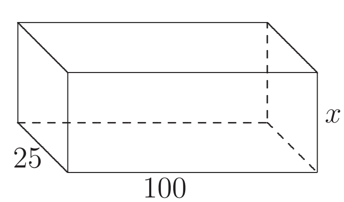
hence (C).

**6.** The total weight of the children is $64 \times 10 = 640$ kg.
The weight of the boys is $4 \times 70 = 280$ kg.
So, the combined weight of the girls is $640 - 280 = 360$ kg.
Hence the average weight of the girls is $360 \div 6 = 60$ kg,

hence (C).

**7.** The length of the call was $623 \div 89 = 7$ minutes, so the call ended at 11:04 am,
hence (C).

**8.** Let the height of the tank be $x$ cm. Now, as $1\,\text{L} = 1000\,\text{cm}^3$, the volume of the tank is $55\,000\,\text{cm}^3$.



The volume of the tank is $25 \times 100 \times x = 55\,000$.

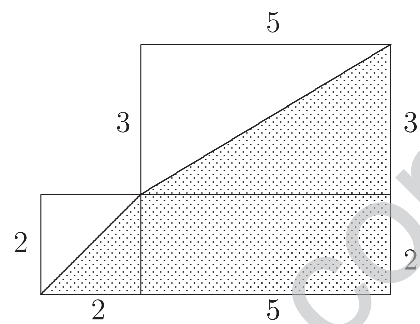
So $x = \dfrac{55\,000}{2500} = 22$,

hence (B).

**9.** Let the smaller number be $x$. Then the larger number is $2x + 3$.
So, $x + 2x + 3 = 18$, $3x = 15$ and $x = 5$,

hence (C).

**10.** (Also J15)
The shaded area is half the $2 \times 2$ square, plus the $2 \times 5$ rectangle, plus half the $3 \times 5$ rectangle.




So the shaded area is $\frac{1}{2} \times 4 + 10 + \frac{1}{2} \times 5 \times 3 = 19.5\,\text{cm}^2$,

hence (D).

**11.** Let the original amount be $x$.
After the first discount of 10%, it becomes $0.9x$
After the second discount of 20%, it becomes $0.8 \times 0.9x$.
After the third discount of 50% it becomes $0.5 \times 0.8 \times 0.9x = 0.36x$.
This means that after the discounts it is 36% of what it was, so the combined discounts are equvalent to 64%,

hence (A).

**12.** (Also S12 & J13)
Using the rules that each row, column and samll square contains 1, 2, 3 and 4, we fill in the squares one at a time, then

| | | | |
|---|---|---|---|
| | 2 | | |
| | | | 1 |
| | 1 | 3 | |
| 4 | | | |

$\longrightarrow$

| | | | |
|---|---|---|---|
| 1 | 2 | | |
| | | | 1 |
| | 1 | 3 | |
| 4 | 3 | | |

$\longrightarrow$

| | | | |
|---|---|---|---|
| 1 | 2 | | |
| | 4 | | 1 |
| 2 | 1 | 3 | |
| 4 | 3 | | |

$\longrightarrow$

| | | | |
|---|---|---|---|
| 1 | 2 | | 3 |
| 3 | 4 | | 1 |
| 2 | 1 | 3 | 4 |
| 4 | 3 | | |

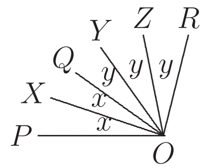
$\longrightarrow$

| | | | |
|---|---|---|---|
| 1 | 2 | 4 | 3 |
| 3 | 4 | 2 | 1 |
| 2 | 1 | 3 | 4 |
| 4 | 3 | 1 | 2 |

The sum of the numbers in the four corners is $1 + 4 + 2 + 3 = 10$,

hence (E).

**13.** Let $\angle POX = \angle QOX = x^\circ$ and $\angle QOY = \angle YOZ = \angle ZOR = y^\circ$.



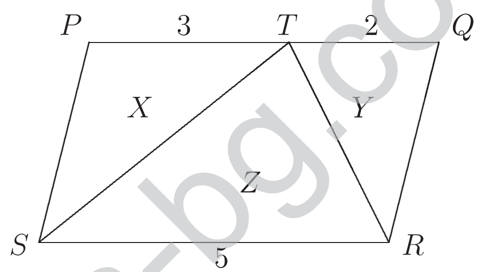

Then $2x + y = 33$ and $x + 2y = 45$.

Solving these simultaneous equations, we get $x = 7$ and $y = 19$.

So, $\angle PQR = x^\circ + x^\circ + y^\circ + y^\circ + y^\circ = 14^\circ + 57^\circ = 71^\circ$,

hence (D).

**14.** (Also S9)
Let the areas of the triangles $PTS$, $TQR$ and $RST$ be $X$,$Y$ and $Z$ respectively.




These triangles have the same height $h$, so $X = \frac{3h}{2}$, $Y = \frac{2h}{2}$ and $Z = \frac{5h}{2}$.
So

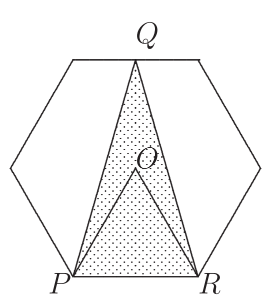
$$\frac{X + Z}{X + Y + Z} = \frac{\frac{3h}{2} + \frac{5h}{2}}{\frac{3h}{2} + \frac{2h}{2} + \frac{5h}{2}} = \frac{4}{5},$$

hence (D).

**15.** The number must divide 45 and must also be greater than 5.
Now, $45 = 3 \times 3 \times 5$, so the possible divisors are $3 \times 3 = 9$, $3 \times 5 = 15$ and $3 \times 3 \times 5 = 45$, that is 3 values,

hence (C).

**16.** If $O$ is the centre of the hexagon, then the area of the hexagon is $6 \times$ area $\triangle POR$.



Now, $\triangle PQR$ has the same base as $\triangle POR$ and twice the height, so the area of $\triangle PQR = 2 \times$ area of $\triangle POR$.
Hence area of the hexagon is $3\,\times$ area of $\triangle PQR$, so the ratio of area $\triangle PQR$ : hexagon is 1 : 3,

hence (B).

*Comment*
It does not matter where the point $Q$ is on the side of the hexagon.

**17.** Let the number of students in the first row be $x$.

Then
the number of students if there are 2 rows is is $2x + 3$,
the number of students if there are 3 rows is is $3x + 9$,
the number of students if there are 4 rows is is $4x + 18$,
the number of students if there are 5 rows is is $5x + 30$,
the number of students if there are 6 rows is is $6x + 45$,
the number of students if there are 7 rows is is $7x + 63$.

Equating each of these to 630,
$x = 630$, possible, but not an alternative,
$2x + 3 = 630$, $2x = 627$, not possible,
$3x + 9 = 630$, $3x = 621$, $x = 207$, possible,
$4x + 18 = 630$, $4x = 612$, $x = 153$, possible,
$5x + 30 = 630$, $5x = 600$, $x = 120$, possible,
$6x + 45 = 630$, $6x = 585$, not possible,
$7x + 63 = 630$, $7x = 567$, $x = 81$, possible.

So, of the alternatives, 6 rows is the only one not possible,

hence (D).



**18.** let the number of runners in the race be $n$ and let Jack finish in $m$th place.

As half as many finished before him as did after him,

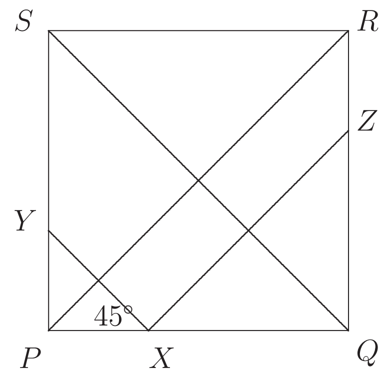
$$\begin{aligned} n - m &= 2(m-1) \\ \therefore \quad n &= 3m - 2. \end{aligned}$$

Jill finished in $(m + 10)$th place and similarly

$$\begin{aligned} m + 9 &= 2(n - (m+10)) \;=\; 2(3m - 2 - m - 10) \\ 3m &= 33\; m = \; 11 \\ \therefore \quad m &= 11 \\ \text{and} \quad n &= 3m - 2 \;=\; 33 - 2 \;=\; 31, \end{aligned}$$

hence (E).

**19.** (Alternative 1)
All the angle are 45° or 90°. Let $YP = PX = a$ and $XQ = QZ = b$, then $a + b = \sqrt{3}$.



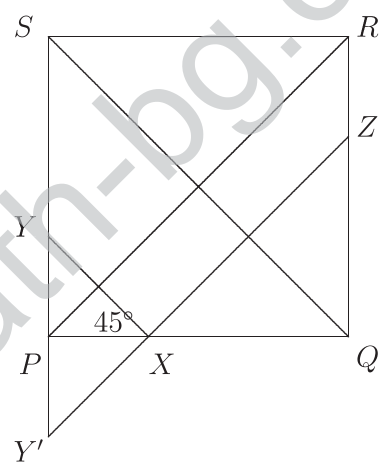

Then, by Pythagoras Theorem, $YX = \sqrt{2}\,a$, $XZ = \sqrt{2}\,b$.
So $YX + XZ = \sqrt{2}(a + b) = \sqrt{6}$,

hence (B).

(ALternative 2)
As $X$ can be any point on $PQ$, move $X$ until it coincides with $P$. Then $XY = 0$ and $XZ = PR = \sqrt{2} \times \sqrt{3} = \sqrt{6}$, so $XY + XZ = \sqrt{6}$,

hence (B).

(Alternative 3)
Reflect $Y$ in $PQ$ to a point $Y'$ on $SP$ extended.






Then $YX + XZ = Y'Z = PR = \sqrt{6}$,

hence (B).

**20.** (Also J25)
We are given the following statements:

1. Andrew says that Bill is a liar.
2. Bill says that Clair is a liar.
3. Clair says that Daniel is a liar.
4. Daniel says that Eva is a liar.

Assume that Andrew is a liar.
Then from 1, Bill must tell the truth.
Then, from 2, Clair must be a liar.
Then, from 3, Daniel must tell the truth.
Then, from 4, Eva must be a liar. This gives 3 liars and 2 who tell the truth.

Assume that Andrew tells the truth.
Then from 1, Bill is a liar.
Then from 2, Clair tells the truth.
Then from 3, Daniel is a liar.
Then from 4, Eva tells the truth.
This gives 2 liars and 3 who tell the truth. So, the maximum number of liars is 3, hence (C).

**21.** (Also J21)
Suppose Rachel was born last century in $19ab$. Then

$$\begin{aligned} 2007-(1900+10a+b) &= 2(1+9+a+b) \\ 107-10a-b &= 20+2a+2b \\ 87 &= 12a+3b \\ 29 &= 4a+b. \end{aligned}$$

Testing possible digits for $a$ gives $a=7, b=1$ or $a=6, b=5$ or $a=5, b=9$. This gives the years 1971, 1965 and 1959 which satisfy the condition.

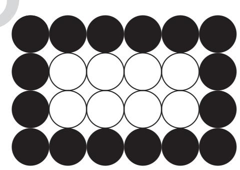
If Rachel was born this century, assume she was born in 200a. Then

$$\begin{aligned} 2007-200a &= 2(2+0+0+a) \\ 7-a &= 4+2a \\ 3a &= 3 \quad \text{and} \quad a = 1. \end{aligned}$$

So 2001 also satisfies as a birthdate, so there are 4 possibles years in which she could have been born,

hence (D).

**22.** Suppose that the array od white counters is $x \times y$, then the array with border is $(x+2)(y+2)$.



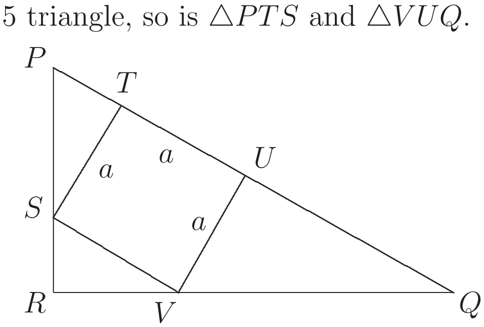
Then

$$\begin{aligned} (x+2)(y+2)-xy &= xy \\ 2x+2y+4 &= xy \\ xy-2x-2y-4 &= 8 \\ (x-2)(y-2) &= 8. \end{aligned}$$

The possibilities are $4 \times 2 = 8$ and $8 \times 1 = 8$.
This gives $x-2=4, y-2=2$ so $x=6, y=4$,
and $x-2=8, y-2=1$ so $x=10, y=3$.
The number of white counters is either 24 or 30,

hence (C).

**23.** Since $PRQ$ is a 3, 4, 5 triangle, so is $\triangle PTS$ and $\triangle VUQ$.



So $PT = \dfrac{3a}{4}$ and $UQ = \dfrac{4a}{3}$. But $PT + TU + UQ = 5$, so

$$\begin{aligned} \frac{3a}{4} + a + \frac{4a}{3} &= 5 \\ 9a + 12a + 16a &= 60 \\ a &= \frac{60}{37}, \end{aligned}$$

hence (D).

**24.** (Also S21 & J30)
(Alternative 1)
There are twelve lift stops altogether. Suppose there are six floors. By the Pigeonhole Principle, some floor has at most two lifts stopping there. Each lift connects this floor to two others, so that only four of the other five floors are connected to this floor. If there are seven or more floors, some floor has at most one lift stopping there, and the situation is worse. Hence there are at most five floors.
This is possible if the first lift stops on floors 1, 4 and 5, the second on 2, 4 and 5, the third in 3, 4 and 5, and the fourth on 1, 2 and 3.

So, the maximum number of floors is 5,

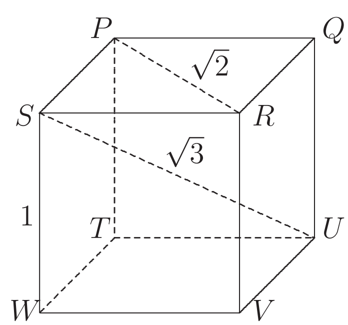
hence (B).

(Alternative 2)
For a building with $n$ floors, there are $T_{n-1}$ pairs which have to be connected, where $T_{n-1}$ is the $n$th triangular number.

Each lift connects 3 pairs (assuming no repeats), so $T_{n=1} \leq 12$.
Now $T_4 = 10$ and $T_5 = 15$, so the largest possible value of $n$ is 5,

hence (B).

**25.** (Also S22)
Consider the cube $PQRSTUVW$.




Since the bee flew so that it visited every vertex of the cube without being twice at the same point, the bee's path consists of exactly 7 straight line segments. The length of an edge is 1 unit, the length of a diagonal of a face is $\sqrt{2}$ and the length of a diagonal of the cube is $\sqrt{3}$.

The bee's path cannot have more than one diagonal of the cube as any two of them meet in the centre of the cube. So the largest possible length of such path is at most $\sqrt{3} + 6\sqrt{2}$. The example $PRUWQTSV$ shows that a path of such length does exist.

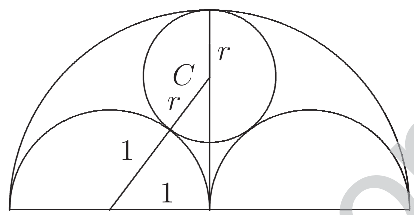
hence (D).

**26.** Using pairs of odd numbers from 1, 2, 3, 4, . . . , 2006, we can get the sum 2008 as follows:
3 + 2005, 5 + 2003, 7 + 2001, . . . , 1003 + 1005.

As $2n - 1 = 1003$, $n = 501$, there are 501 such pairs, and these pairs use all the odd integers in the range other than 1.

So, the maximum number of odd numbers we can choose so that they no not add to 2008 is 502, as we can choose 1 and then one odd number from each of the 501 pairs.
So, the minimum number we must choose to ensure that two of them add to 2008 is 503.

**27.** Let the radius if circle $C$ be $r$. Join the centres of the two semicircles and the circle $C$ as shown.




Then, from the right-angled triangle, we get

$$\begin{aligned} 1^2 + ((2-r)^2 &= (1+r)^2 \\ 1 + 4 - 4r + r^2 &= 1 + 2r + r^2 \\ 6r &= 4 \\ r &= \frac{2}{3} = \frac{a}{b} \end{aligned}$$

Hence $a + b = 5$.

**28.** (Also S28)
Let $10a + b$ be a number with at most two digits.

The equation $10a + b = 19(a + b)$ cannot hold unless $a = b = 0$. So all lucky numbers have at least three digits.

Suppose a lucky number has $m$ digits for some $m \geq 4$. Then its digit sum is at most $9m$ while the number is at least $10^{m-1}$. Hence $171m \geq 10^{m-1}$.

For $m = 4$, $684 \geq 1000$ is false, so there are no lucky numbers with 4 digits.

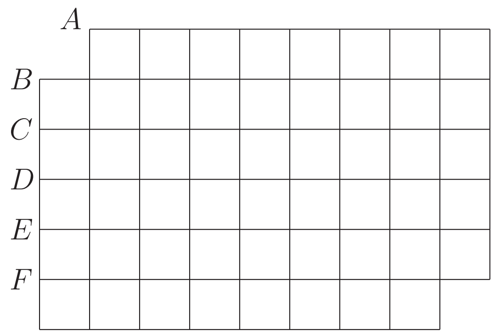
For $m \geq 5$, the situation is worse. Hence all lucky numbers have exactly three digits. Suppose the number is $abc$.

Then $100a + 10b + c = 19a + 19b + 19c$, we have $81a = 9b + 18c$ or $9a = b + 2c$.

For $a = 1$, we have $(b, c) = (1, 4)$, (3,3), (5,2), (7,1) and (9,0).
For $a = 2$, we have $(b, c) = (0, 9)$, (2,8), (4,7), (6,6) and (8,5).
For $a = 3$, we have $(b, c) = (9, 9)$, and there are no other solutions.

Hence there are exactly 11 lucky numbers, namely, 114, 133, 152, 171, 190, 209, 228, 247, 266, 285 and 399.

**29.** (Also J29)
(Alternative 1)
Label the points $A, B, C, D, E$ and $F$ as shown.




Consider the $1 \times 1$ squares;
Across from $A$ there are 8; from $B$ there are 9, from $C$ there are 9, from $D$ there are 9, from $E$ there are 9, from $F$ there are 8; giving 52 $1 \times 1$ squares.

Consider the $2 \times 2$ squares:
Across from $A$ there are 7, from $B$ there are 8, from $C$ there are 8, from $D$ there are 8 and from $E$ there 7, giving 38 $2 \times 2$ squares.

Consider the $3 \times 3$ squares:
Across from A there are 6, from $B$ there are 7, from $C$ there are 7 and from $D$ there are 6, giving 26.

Consider the $4 \times 4$ squares:
Across from $A$ there are 5, from $B$ there are 6 and from $C$ there are 5, giving 16 $4 \times 4$ squares.

Consider the $5 \times 5$ squares:
Across from $A$ there are 3, from $B$ there are 4, giving 8 $5 \times 5$ squares.

Consider the $6 \times 6$ squares, there are 2 across from $A$ and no others.

The total number of squares of all sizes is then $52 + 38 + 26 + 16 + 8 + 2 = 142$.

(Alternative 2)
Add in the two missing squares.

Then for the $1 \times 1$ squares there are $6 \times 9 = 54$,
for the $2 \times 2$ squares there are $5 \times 8 = 40$,
for the $3 \times 3$ squares there are $4 \times 7 = 28$.
for the $4 \times 4$ squares there are $3 \times 6 = 18$,
for the $5 \times 5$ squares there are $2 \times 5 = 10$,
for the $6 \times 6$ squares there are $1 \times 4 = 4$.

This gives a total of 154 squares.

Now take away the squares which include either of the two corners, which is 2 in each case so a total of $2 \times 6 = 12$, and the number of sqaures is $154 - 12 = 142$.

**30.** (Also S29)
The digits in base 10 which can be read upside down are 0, 1, 2, 5, 6, 8 and 9.

Then, writing numbers which can be read upside down is like writing numbers in base 7 using just those digits.

Writing 2007 in base is $5 \times 7^3 + 5 \times 7^2 + 6 \times 7 + 5 = 5565$.

But, in this pseudo base 7, the 5 is replaced by 8 and the 6 by 9.

So, 5565 is written as 8898 and is the 2007th number to be read upside down. The last three digits are 898.

* * *