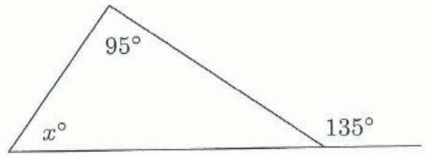
# Solutions – Intermediate Division

**1.** $92.2 - 85.3 = 6.9$,

hence (B).

**2.** The angle $135°$ is the exterior angle of the triangle and is equal to the sum of the two marked angles.




So $x + 95 = 135$ and $x = 40$,

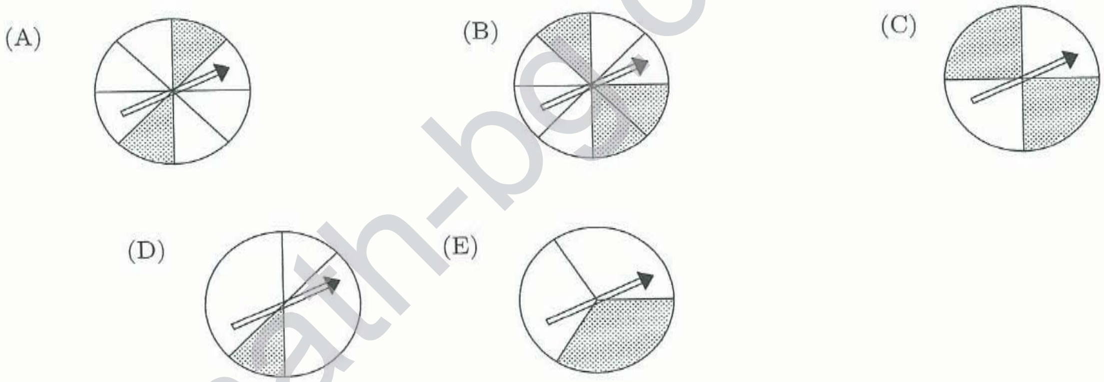
hence (B).

**3.** (Also S2)

$$\begin{aligned} a &= 2b - 5 \\ 2b &= a + 5 \\ b &= \frac{a+5}{2}, \end{aligned}$$

hence (D).

**4.** (Also J11)
Spinner (A) has $\frac{2}{8} = \frac{1}{4}$ shaded, spinner (B) has $\frac{3}{8}$ shaded,




spinner (C) has $\frac{2}{4} = \frac{1}{2}$ shaded, spinner (D) has $\frac{1}{8}$ shaded and spinner (E) has $\frac{1}{3}$ shaded, so the spinner with a 1 in 4 chance is (A),

hence (A).

**5.** (Also J8)
Since $64 = 4 \times 4 \times 4$, the length of the edge of a face is 4 cm, and the area of a face in square centimetres is $4 \times 4 = 16$,

hence (B).

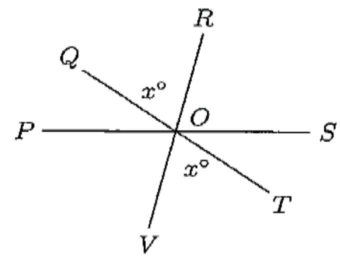
**6.** (Also J10)
The 5 numbers are averaged, so their sum is $5 \times 4 = 20$. Since $1 + 2 + 3 + 4 = 10$ it follows that the fifth number is 10,

hence (E).

**7.** $\frac{1}{4}\% = 0.25 \div 100 = 0.0025$,

hence (E).

**8.** (Also J15 & S3)
In the diagram, $\angle POR = 120°$ and $\angle QOS = 145°$.




Let $\angle TOV = \angle QOR = x°$.
Then $\angle POR + \angle QOS = \angle POS + x° = 180° + x°$.
So $120 + 145 = 180 + x$ and $x = 85$,

hence (C).

**9.** (Also S6)
If you begin reading at the top of page 13 and finish at the bottom of page 14 you have read $14 - 13 + 1 = 2$ pages, so if you start at the top of page $x$ and read to the bottom of page $y$ you will have read $y - x + 1$ pages,

hence (D).

**10.** The first possibility for the second last digit is 4, as 2 and 3 already occur, so the next reading with all digits different is 062341, a distance of 22 kilometres,

hence (C).

**11.** (Also J16)
There are 365 days in the year 2006.
Thus the middle day of the year is the 183rd day of the year.
January has 31 days, February 28, March 31, April 30, May 31 and June 30, giving a total of 181 days in the first 6 months, so the middle date is 2 days on, on 2nd July,

hence (D).

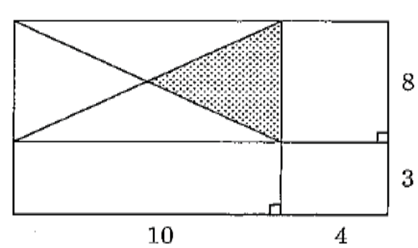
**12.** The sums for $PQ$ and $RS$ add up to the sum of all four numbers given to the vertices of the square $PQRS$. Similarly, the sums for $QR$ and $PS$ add up to the sum of all four numbers given to the vertices of the square $PQRS$.
Hence the sum for $PS$ equals $3 + 12 - 7 = 8$,

hence (C).

**13.** In the sequence of numbers $\ldots, q, r, s, t, 0, 1, 1, 2, 3, 5, 8, \ldots$, each number is the sum of its two preceeding numbers.
So $t = 1$, since $1 + 0 = 1$;
$s = -1$, since $-1 + 1 = 0$;
$r = 2$, since $2 - 1 = 1$;
$q = -3$, since $-3 + 2 = -1$,

hence (A).

**14.** (Also S9)
The shaded area is $\frac{1}{4} \times 8 \times 10 = 20$ square units (quarter of the rectangle).



The total area is $14 \times 11$ square units.
The fraction shaded is then $\frac{20}{14 \times 11} = \frac{10}{7 \times 11} = \frac{10}{77}$,

hence (E).

**15.** (Also S10)
The train takes $\frac{1}{4}$ of a minute to pass a post and $\frac{3}{4}$ of a minute to pass through a 600 m tunnel. This means that it takes the front of the train $\frac{1}{2}$ of a minute to pass through the 600 m tunnel and that the train is travelling 0.6 km every $\frac{1}{2}$ minute and so is travelling at $0.6 \times 120 = 72$ km/h,

hence (D).

**16.**

$$\begin{array}{ccccc} & P & 7 & * & * \\ & & & & 6 \times \\ \hline * & 2 & * & 8 & 4 \end{array}$$

The carry into the thousands ($P$) column from the hundreds column (7) must be either 4 or 5. But it cannot be 5, since $6P$ and 2 are both even.
Therefore the carry is 4 and $6P$ is a number whose units digit is 8.
Then $P = 3$ or 8, but 3 is not offered as a choice,

hence (E).

*Comment*
Though it is not necessary, we could also have worked from the right and found that there are four different posibilities:-
$3714 \times 6 = 22284$, $3764 \times 6 = 22584$, $8714 \times 6 = 52284$ and $8764 \times 6 = 52584$, and so $P = 3$ or 8 as before.



**17.** (Also J24 & S15)
If a 3-digit number has digits all the same then the number is one of the numbers 111, 222, 333, ... , 999.

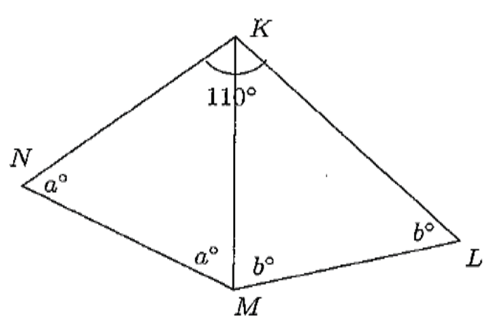
Each of these numbers is divisible by 111 and the prime factors of 111 are 3 and 37.

So, one of the two-digit numbers must be a multiple of 3 and the other must be a multiple of 37.

We cannot obtain the products 111, 222 or 333 multiplying two two-digit numbers. We can get
$(1 \times 37) \times (4 \times 3) = 444$,
$(1 \times 37) \times (5 \times 3) = 555$,
$(1 \times 37) \times (6 \times 3) = 666$,
$(1 \times 37) \times (7 \times 3) = 777$,
$(1 \times 37) \times (8 \times 3) = 888$,
$(1 \times 37) \times (9 \times 3) = 999$,
$(2 \times 37) \times (4 \times 3) = 888$,
with 888 being the only one obtainable in two ways, so the number of pairs is 7,

hence (C).

**18.** Since $KM = KL = KN$, $\angle KNM = \angle KMN = a°$ and $\angle KML = \angle KLM = b°$




From the angle sum of the triangles $KNM$ and $KLM$ we get

$$2a + 2b + 110 = 360, \quad a + b = 125,$$

so $\angle NML = a + b = 125°$,

hence (C).

**19.** Now as $a \oplus b = \dfrac{b}{a} - 1$, we get

$$\begin{aligned}(3 \oplus 4) \oplus (1 \oplus 2) &= \left(\frac{4}{3} - 1\right) \oplus \left(\frac{2}{1} - 1\right) \\ &= \frac{1}{3} \oplus 1 \\ &= \frac{1}{\frac{1}{3}} - 1 = 3 - 1 = 2,\end{aligned}$$

hence (B).

**20.** (Also S16)
Let the amount of salt in the original mixture be $x$ grams. This means the fraction of salt in the mix is $\dfrac{x}{450}$. When this saltiness is reduced by 10% by adding $y$ litres of flour, this fraction becomes $\dfrac{9}{10} \times \dfrac{x}{450}$ and so

$$\begin{aligned}\frac{x}{450 + y} &= \frac{9}{10} \times \frac{x}{450} \\ &= \frac{x}{500},\end{aligned}$$

so 50 grams of flour must be added,

hence (A).

**21.** Now $72 = 2^3 \times 3^2$, so all factors of 72 are multiples of 2 or 3. So if the number $N$ and 72 have no common factors, then the number of occurrences of $N$ ($1 \le N < 72$) is

= 71 − # multiples of 2 − # multiples of 3 + # multiples of 2 and 3

$= 71 - 35 - 23 + 11 = 24$,

hence (E).

**22.** (Also J23 & S20)
(Alternative 1)
For each of the three choices to place R in row one, there are two choices to place R in row two and then only one choice in row 3.
So, there are 6 ways of placing the Rs.

| | | |
|---|---|---|
| R | | |
| | | R |
| | R | |

For each of these, there are two choices to place W in row one, then one in row two and one in row three, and for each of these, the placement of B is determined.
Hence the number of different patterns is $6 \times 2 \times 1 = 12$,

hence (D).

(Alternative 2)
There are 6 ways of placing 3 colours in the top row.
Then there are 2 choices for colours in the second row and the final row is then determined, so there are $6 \times 2 = 12$ ways,

hence (D).

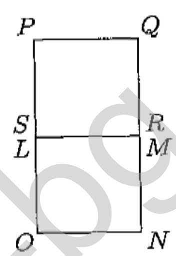
**23.** (Also S17)
Let the weights of the bales in kilograms be $a, b, c, d, e$. Then as all pairs of weights are different, the weights of the five bales are different. Assume then that $a < b < c < d < e$.
The lowest two sums have to be $a + b$ and $a + c$ and the highest two sums have to be $c + e$ and $d + e$.
Also, as each bale is weighed in pairs with four others, the sum of the weights of all pairs must be 4 times their combined weights, so $4(a+b+c+d+e) = 110 + 112 + 113 + 114 + 115 + 116 + 117 + 118 + 120 + 121 = 1156$ and $a + b + c + d + e = 289$.
So we have five equations

$$\begin{aligned} a+b &= 110 \qquad (1)\\ a+c &= 112 \qquad (2)\\ c+e &= 120 \qquad (3)\\ d+e &= 121 \qquad (4)\\ a+b+c+d+e &= 289 \qquad (5) \end{aligned}$$

Substituting (1) and (4) in (5) gives $110 + c + 121 = 289$ and so $c = 58$.
Substituting this in (3) gives $e = 62$ so the heaviest bale is 62 kg,

hence (E).

**24.** (Also S21)
$P$ moves along 3 circular arcs. The first rotation about $R$ is 180° with a radius of the diagonal of the square, $\sqrt{2}$. This arc is then of length $\pi \times \sqrt{2}$.

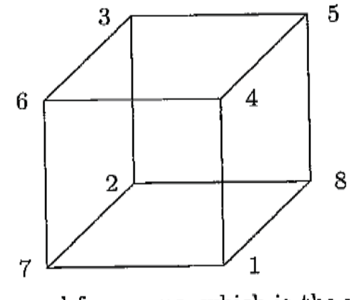



The next rotation about $Q$ is 180° with radius 1, so the length is $\pi$.
The third rotation is about $P$ so $P$ does not move.
The last rotation is 180° about $L$ with radius 1 so has length $\pi$.
The total length of the path traced out is then

$$\pi \times \sqrt{2} + \pi + \pi = \pi(2 + \sqrt{2}),$$

hence (A).

**25.** (Also J29 & S22)
There are 6 faces and hence 6 face sums. Since each vertex lies on 3 faces, the sum of all the face sums of the cube is $3(1+2+3+4+5+6+7+8) = 108$, so if there are 6 equal face sums, that sum must be $108 \div 6 = 18$.




The figure gives an example of 6 equal face sums, which is the maximum number possible.

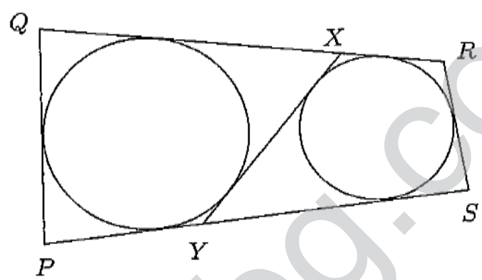
**26.** Given $(1+3+5+\cdots+p)+(1+3+5+\cdots+q)=(1+3+5+\cdots+25)$, using the formula for the sum of an arithmetic progression, we get

$$\begin{aligned}\frac{p+1}{4}(1+p)+\frac{q+1}{4}(1+q) &= \frac{26}{4}\times(26)\\ \left(\frac{p+1}{2}\right)^2+\left(\frac{q+1}{2}\right)^2 &= \left(\frac{26}{2}\right)^2=13^2\end{aligned}$$

Since $5^2+12^2=13^2$ and $\frac{p+1}{2}$ and $\frac{q+1}{2}$ must both be positive integers, $\frac{p+1}{2}$ and $\frac{q+1}{2}$ must be 5 and 12 in some order. So $p$ and $q$ are 9 and 23 in some order, so $p+q=32$.

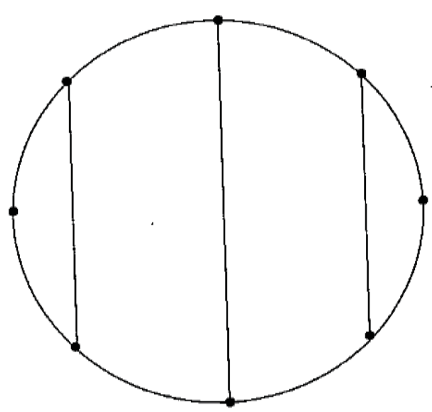
**27.** (Also J27 & S26)
There are 18 possible digit sums for two-digit numbers: 1, 2, 3, ..., 17, 18.
There is only one two-digit number with digit sum 1, namely 10; and there is only one two-digit number with digit sum 18, that is 99.
Therefore the largest number of two-digit numbers that can be written on the whiteboard, such that no three numbers have the same digit sum, is $1\times 2+2\times 16=34$.
Thus the smallest number of students in the class for the teacher to be correct is 35.

**28.**

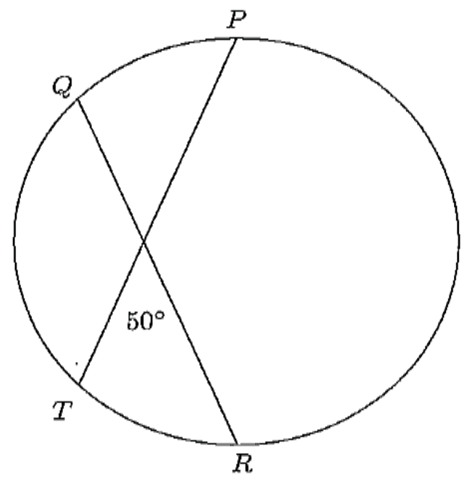



Since a circle touches all four sides of $PQXY$, we have $PQ+XY=QX+PY$. (This follows from the property that tangents to a circle from an external point are equal.)
Similarly, since a circle touches all four sides of $XRSY$, we have $RS+XY=RX+SY$.
Adding yields $PQ+RS+2XY=QR+PS$.
Hence $2XY=QR+PS-PQ-RS=20+26-10-14=22$ and $XY=11$.

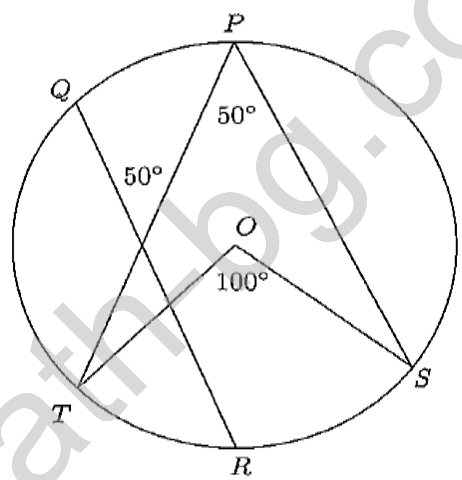
**29.** (Also S28)
(Alternative 1)
The question does not stipulate that the two diagonals must share a common vertex. However, for each diagonal in a regular polygon with more than 5 sides, there are parallel diagonals through other vertices. An example follows:



Consider the case where a pair of diagonals $PT$ and $QR$ intersect at an angle of $50°$ as in the following diagram.




Then, if arc $PQ <$ arc $PT$, then there must exist a point $S$ on arc $PT$ such that $PS \parallel QR$. So $TS$ subtends an angle of $50°$ at the circumference of the circle and also an angle of $100°$ at the centre.



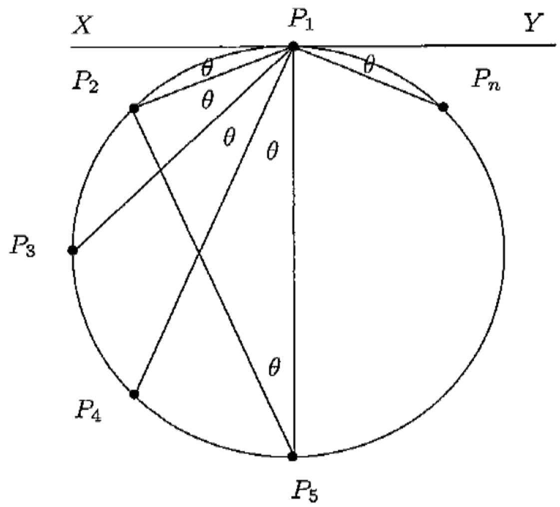



It follows that the angle subtended at the centre of the circle by a single side of the polygon must be a divisor of $100°$ and also of $360°$. The highest common factor of 100 and 360 is 20, so the smallest number of edges the polygon can have is $\dfrac{360}{20} = 18$.

(Alternative 2)
For the angle between two diagonals in a regular $n-$gon to be $50°$, there must be two vertices of this polygon such that they divide the perimeter of the polygon in ratio 50 : 130.
Hence there exists a postive integer $a$ such that $\frac{a}{n-a} = \frac{5}{13}$.
Therefore $13a = 5n - 5a$, and $18a = 5n$. Since 5 and 18 are relatively prime, $n$ must be divisible by 18. So $n \geq 18$ and the example of $n = 18$, $a = 5$ shows that $n = 18$ is possible.

Hence the smallest value of $n$ is 18.

(Alternative 3)
Inscribe the polygon $P_1, P_2, P_3, \ldots$ in a circle. Let $XY$ be the tangent at $P$.




Angle $P_2P_1P_3$, angle $P_3P_1P_4$ and so on are equal ($\theta$) as they are subtended by equal arcs at $P_1$.

Angle $XP_1P_2$ and angle $YP_1P_n$ are also equal (to $\theta$) by the alternate segment theorem.

Hence $\angle XP_1Y$ is divided into $n$ equal angles by edges $P_1P_2$, $P_1P_n$ and by all diagonals drawn from $P_1$ to the other $n-1$ vertices of the polygon.

Each of these $n$ angles is equal to $\dfrac{180^\circ}{n}$.

All angles between diagonals are integer multiples of this basic angle as moving from $P_1$ to $P_2$ rotates all diagonals by $\frac{360}{n} = 2 \times \frac{180}{n}$, so that each diagonal is parallel to a diagonal, edge or tangent at $P_1$ and this applies to each vertex.

So, to generate $50^\circ$ between diagonals, we need to find the smallest value of $n$ which makes $\frac{180}{n}$ a factor of 50.

The highest common factor of 180 and 50 is 10, so $n$ is minimum when $\frac{180}{n} = 10$ and this is when $n = 18$.

**30.** (Also S29)
We are looking for a maximum product

$$n_1 \times n_2 \times n_3 \times \ldots \times n_k$$

where $n_1 + n_2 + n_3 + \cdots + n_k = 19$.

If any factor $n_i$ is $\geq 5$, it can be replaced by two factors 2 and $n_i - 2$ which leave the sum unchanged, but increases the product since $2 \times (x-2) > x$ for $x \geq 5$. So, every factor in the largest product is $\leq 4$.

Similarly, if any factor $n_i$ is equal to 4, it can be replaced by $2 \times 2$ with no change to the product, so we shall do this and then every factor is $\leq 3$.
If any factor is 1, it can be combined with another factor, replacing $1 \times n_i$ by $(n_i + 1)$ which increases the product, so now all factors in the largest product are 2 or 3.
If there are three or more 2s, $2 \times 2 \times 2$ can be replaced by $3 \times 3$ to increase the product. So in the largest product, there are at most two 2s.
There is only one way that 19 can be written as such a sum: there are five 3s and two 2s.
So the maximum product is $3^5 \times 2^2 = 972$.