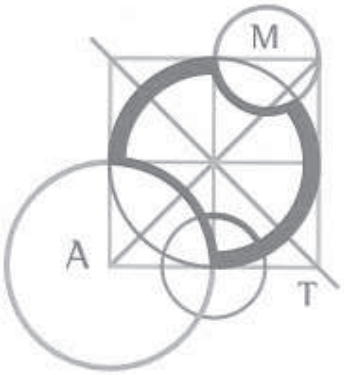
Westpac

AUSTRALIAN MATHEMATICS COMPETITION
FOR THE WESTPAC AWARDS

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST



THURSDAY 27 JULY 2006

INTERMEDIATE DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 9 AND 10
TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple choice questions, each with 5 possible answers given and 5 questions that require a whole number between 0 and 999. The questions generally get harder as you work through the paper.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the **Answer Sheet** carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only pencil.
2. Record your answers on the reverse of the Answer Sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMC reserves the right to re-examine students before deciding whether to grant official status to their score.

Intermediate Division

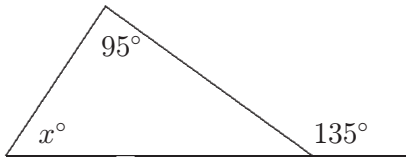
Questions 1 to 10, 3 marks each

1. $92.2 - 85.3$ equals

(A) 6.1 (B) 6.9 (C) 7.1 (D) 7.5 (E) 7.9

2. In the diagram, the value of x is

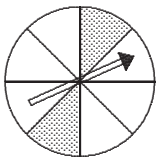
(A) 35 (B) 40 (C) 45 (D) 50 (E) 55

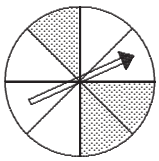


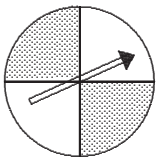
3. If $a = 2b - 5$, then b equals

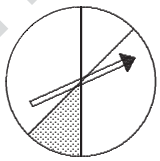
(A) $\frac{a}{2}$ (B) $\frac{a}{2} + 5$ (C) $\frac{a-5}{2}$ (D) $\frac{a+5}{2}$ (E) $2a + 5$

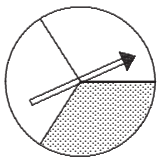
4. Which of the spinners below would give a one-in-four chance of the arrow landing in the shaded region?

(A) 

(B) 

(C) 

(D) 

(E) 

5. The area, in square centimetres, of one face of a cube whose volume is 64 cm^3 is

(A) 8 (B) 16 (C) 24 (D) 32 (E) 64

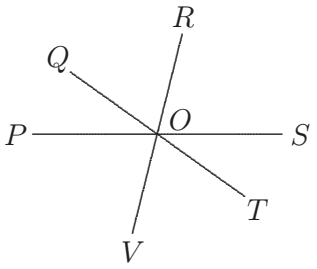
6. The average of five numbers is 4. Four of them are 1, 2, 3 and 4. What is the other?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

7. $\frac{1}{4}\%$ expressed as a decimal is

(A) 0.235 (B) 0.14 (C) 0.025 (D) 0.014 (E) 0.0025

8. In the diagram, $\angle POR = 120^\circ$ and $\angle QOS = 145^\circ$. The size of $\angle TOV$ is



(A) 45° (B) 60° (C) 85° (D) 90° (E) 95°

9. The page numbers of a book are consecutive whole numbers. If you begin reading at the top of page x and stop reading at the bottom of page y, the number of pages you have read is

(A) $x - y$ (B) $y - x$ (C) $x + y$ (D) $y - x + 1$ (E) $y - x - 1$

10. Jim notices the odometer of his car reads 062319 km where all the digits are different. The shortest distance, in kilometres, he will travel before the digits are all different again is

(A) less than 10 (B) between 10 and 20 (C) between 20 and 30 (D) between 30 and 40 (E) greater than 40

Questions 11 to 20, 4 marks each

11. The middle date of the year in 2006 is

(A) 29th June (B) 30th June (C) 1st July (D) 2nd July (E) 3rd July

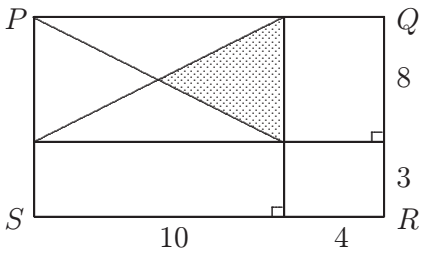
12. Each of the vertices of a square $PQRS$ is given a number. For each of the sides of the square the sum of the numbers at its endpoints is calculated. If for PQ this sum is 3, for QR it is 7 and for RS it is 12, what is the sum for PS?

(A) 2 (B) 7 (C) 8 (D) 16 (E) 22

13. In the sequence of numbers $\ldots, q, r, s, t, 0, 1, 1, 2, 3, 5, 8, \ldots$, each number is the sum of its two preceeding numbers. What is the value of q?

(A) -3 (B) -1 (C) 0 (D) 1 (E) 3

14. What fraction of the rectangle $PQRS$ in the diagram is shaded?



(A) $\frac{1}{16}$ (B) $\frac{3}{5}$ (C) $\frac{1}{8}$ (D) $\frac{1}{10}$ (E) $\frac{10}{77}$

15. A train travelling at constant speed takes a quarter of a minute to pass a signpost and takes three-quarters of a minute to pass completely through a tunnel which is 600 m in length. The speed of the train, in kilometres per hour, is

(A) 50 (B) 56 (C) 64 (D) 72 (E) 80

16. In the multiplication

$$\begin{array}{cccc} P & 7 & * & * \\ & & & 6 \quad \times \\ \hline * & 2 & * & 8 \quad 4 \end{array}$$

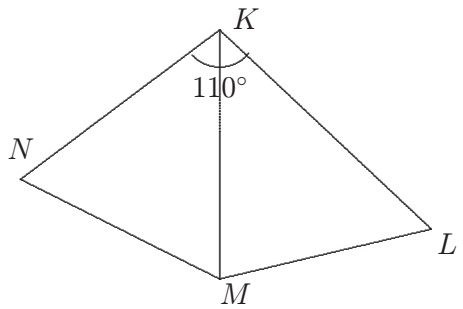
P and $*$ stand for digits. P could be

(A) 7 (B) 6 (C) 5 (D) 9 (E) 8

17. How many different pairs of 2-digit numbers multiply to give a 3-digit number with all digits the same?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

18. In the quadrilateral $KLMN$, $KM = KL = KN$.
If $\angle NKL = 110°$, then the size of $\angle LMN$ is



(A) 70° (B) 115° (C) 125° (D) 140° (E) 145°

I 4

19. Define the operation $\oplus$ as $a \oplus b = \dfrac{b}{a} - 1$. The value of $(3 \oplus 4) \oplus (1 \oplus 2)$ is

(A) 0 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) 5

20. I have 450 grams of salt and flour mix. How many grams of flour should I add to reduce the percentage of salt in the mixture to 90% of what it was?

(A) 50 (B) 10 (C) 30 (D) 45 (E) 60

Questions 21 to 30, 5 marks each

21. How many positive integers less than 72 have the property that the highest common factor of the number and 72 is equal to 1?

(A) 12 (B) 30 (C) 36 (D) 18 (E) 24

22. The nine squares of a 3×3 grid painted on a wall are to be coloured red, white and blue so that no row or column contains squares of the same colour. One such pattern is shown in the diagram. How many different patterns can be made?

R	W	B
B	R	W
W	B	R

(A) 15 (B) 6 (C) 9 (D) 12 (E) 24

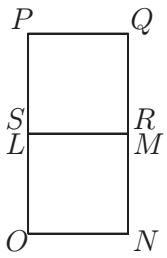
23. Five bales of hay are weighed two at a time in all possible combinations. The weights, in kilograms, are:-

110, 112, 113, 114, 115, 116, 117, 118, 120 and 121.

What is the weight, in kilograms, of the heaviest bale?

(A) 58 (B) 59 (C) 60 (D) 61 (E) 62

24. The squares $PQRS$ and $LMNO$ have equal sides of 1 m and are initially placed so that the side SR touches LM as shown. The square $PQRS$ is rotated about R until Q coincides with N. The square is then rotated about Q until P coincides with O.
It is then rotated about P until S coincides with L and then finally rotated about S until R coincides with M and the square is now back to its original position.



The length, in metres, of the path traced out by the point P in these rotations is

(A) $\pi(2+\sqrt{2})$ (B) 4π (C) $2\pi(2+\sqrt{2})$ (D) 2π (E) $\pi(3+\sqrt{2})$

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25. The vertices of a cube are each labelled with one of the integers 1, 2, 3, ..., 8. A *face-sum* is the sum of the labels of the four vertices on a face of the cube. What is the maximum number of equal face-sums in any of these labellings?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

26. If $(1+3+5+\cdots+p)+(1+3+5+\cdots+q)=(1+3+5+\cdots+25)$,
what is the value of $p+q$?

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27. Each of the students in a class writes a different 2-digit number on the whiteboard. The teacher claims that no matter what the students write, there will be at least three numbers on the whiteboard whose digits have the same sum. What is the smallest number of students in the class for the teacher to be correct?

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28. In a quadrilateral $PQRS$, X is a point on QR and Y is point on PS. One circle touches all four sides of the quadrilateral $PQXY$, and another circle touches all four sides of $XRSY$. If $PQ = 10\,\text{cm}$, $QR = 20\,\text{cm}$, $RS = 14\,\text{cm}$ and $PS = 26\,\text{cm}$, what is the length, in centimetres, of XY?

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29. In a regular polygon there are two diagonals such that the angle between them is $50°$. What is the smallest number of sides of the polygon for which this is possible?

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30. The sum of n positive integers is 19. What is the maximum possible product of these n numbers?
