Solutions – Junior Division

1. $2.6 + 0.12 = 2.72$,

hence (E).

2. $1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30$,

hence (B).

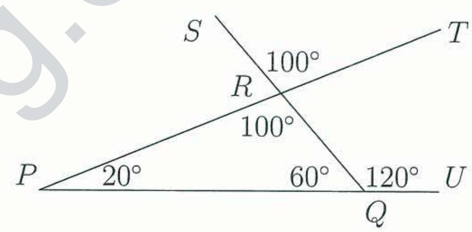
3. 6:35 pm is 35 minutes before 7:10 pm,

hence (A).

4. \$10 in 10c coins is $10 \times 10 = 100$ coins.
\$10 in 20c coins is $10 \times 5 = 50$ coins.
\$10 in 50c coins is $10 \times 2 = 20$ coins.
The total number of coins is $100 + 50 + 20 = 170$,

hence (E).

5. We have $\angle RPQ = 20°$ and $\angle RQU = 120°$.
So $\angle PQR = 60°$ and $\angle PRQ = \angle SRT = 100°$,



hence (D).

6. (Also I1)
$(2000 + 9) + (2000 - 9) = 4000 + 9 - 9 = 4000$,

hence (A).

7. The line is a rectangle with width 0.5 mm and area of 1 square metre. So, if its length is l m, $0.0005 \times l = 1$ and $l = \dfrac{1}{0.0005} = \dfrac{10\,000}{5} = 2000$,

hence (D).

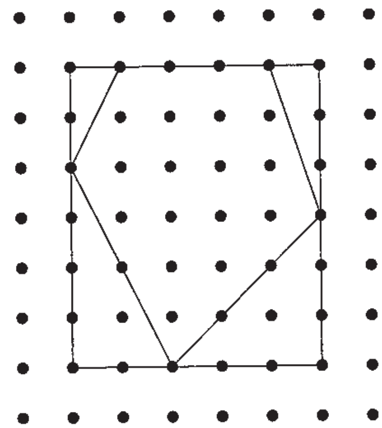
8. *Alternative 1*
The time needed will be $\dfrac{91}{26} \times 2 = \dfrac{7}{2} \times 2 = 7$ hours,

hence (B).

Alternative 2
26 tests take 2 hours, so 13 tests will take 1 hour and so 91 tests will take 7 hours,

hence (B).

9. Construct a rectangle around the figure.



The area of the figure is the area of the rectangle $5 \times 6 = 30$ less the four triangles on the corners, that is, $30 - \frac{1}{2} \times 2 \times 1 - \frac{1}{2} \times 3 \times 1 - \frac{1}{2} \times 3 \times 3 - \frac{1}{2} \times 2 \times 4$ $= 30 - 1 - 1\frac{1}{2} - 4\frac{1}{2} - 4 = 19$ square centimetres,

hence (B).

10. (Also I4)
(A) is $\frac{1}{3}$, (B) is $\frac{2}{3}$, (C) is $\frac{1}{9}$, (D) is 0 and (E) is 1, so the largest is 1,

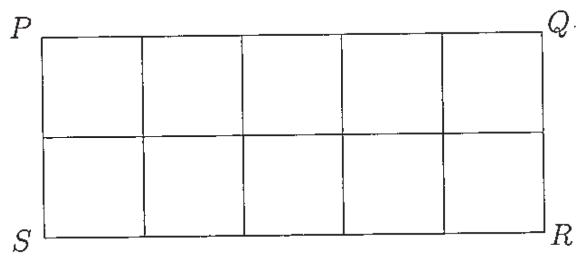
hence (E).

11. (Also I5)
We have

$$\begin{aligned} 0.1 \times 0.2 \times 0.3 \times 0.4 \times \square &= 0.12 \\ 0.0024 \times \square &= 0.12 \\ \square &= \frac{0.12}{0.0024} = \frac{1200}{24} = 50, \end{aligned}$$

hence (B).

12. (Also I10)
Let the side length of a square be s. Then the perimeter of the rectangle is $5s + 2s + 5s + 2s = 14s$.



So $14s = 21$ and $s = \frac{21}{14} = \frac{3}{2} = 1.5$ cm, so the perimeter of the square is $4 \times 1.5 = 6$ cm,

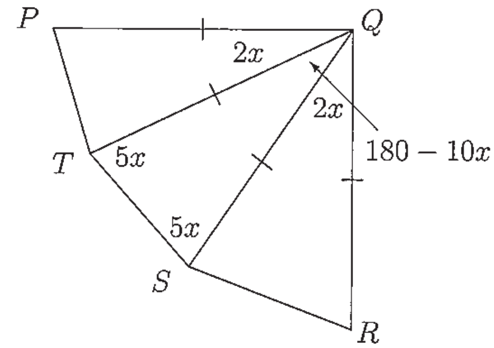
hence (C).

13. Now $1 \times 3 \times 5 \times 7 \times 9 = 3 \times 63 \times 5$ which is an odd multiple of 5, so the last digit is 5. Each group of five numbers in the product $1 \times 3 \times 5 \times \cdots \times 997 \times 999$ when multiplied together ends in 5, so the product of all 500 numbers ends in 5,

hence (C).

14. (Also I12)
With all angles in degrees, in the triangle QST, $\angle QST = 5x$ and then $\angle SQT = 180 - 10x$.
Now $\angle PQR = 90$, so
$(180 - 10x) + 4x = 90$, $6x = 90$ and $x = 15$,

hence (D).



15. If half the class do both and the number who swim is the same as the number who cycle, then $\frac{1}{4}$ of the students swim only and $\frac{1}{4}$ cycle only. Then $\frac{3}{4}$ of the students swim, and $\frac{3}{4}$ of the class is 24 students, so the class consists of 32 students,

hence (C).

16. The difference in length between the \$10 note and the \$100 note is $3 \times 7 = 21$ mm. The difference in area is $21 \times 65 = 1365$ square millimetres,

hence (C).

17. Since the sum of the numbers in the diagonal is the same as the sum of the column on the right, we get
$8 + x + y = y + 10 + 12$,
so $x = 22 - 8 = 14$.
This tells us that the diagonal, row and column sum is 42.
So $8 + x + y = 42$ and $x + y = 34$,

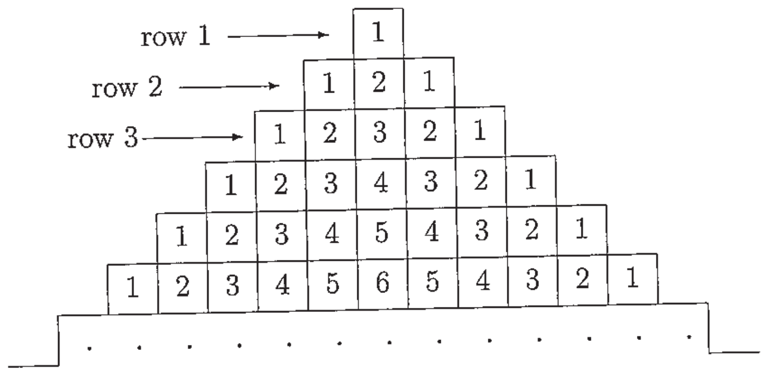
16		y
	x	10
8		12

hence (A).

18. Let the train from Canberra be at a point P when the other train leaves Sydney. The train from Sydney will reach P at 3:30 pm (which is 170 minutes later) when the first train arrives at Sydney. Since both trains travel at the same speed, they will pass at the midpoint between P and Sydney, 85 minutes after 12:40 pm, which is 2:05 pm,

hence (C).

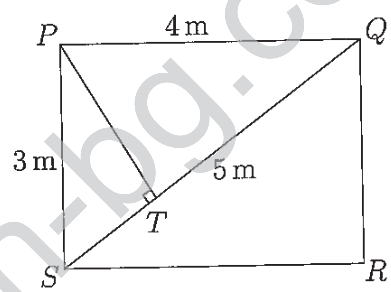
19. The middle number in the 57th row is 57, so there are $57 + 56 = 113$ numbers in the 57th row.



We want the 83rd number from the left which has $113 - 83 = 30$ numbers to its right, so is 31,

hence (B).

20. The area of $\triangle PQS = \frac{1}{2} \times 3 \times 4 = \frac{1}{2} \times QS \times PT$. So $5 \times PT = 12$.



Then $PT = \dfrac{12}{5} = 2.4$,

hence (D).

21. Consider the primes 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, ...
Since $11 \times 13 > 100$, we only need to consider the multiples of 2, 3, 5 and 7.
$2 \times 5, 2 \times 7, \ldots, 2 \times 47$ gives thirteen 2-digit numbers.
$3 \times 5, 3 \times 7, \ldots, 3 \times 31$ gives nine 2-digit numbers.
$5 \times 7, 5 \times 11, \ldots, 5 \times 19$ gives five 2-digit numbers.
$7 \times 11, 7 \times 13$ gives two 2-digit numbers.
The total number of such 2-digit numbers is $13 + 9 + 5 + 2 = 29$,

hence (E).

22. *Alternative 1*
Let the number of fish which Billy, Lenny and Peter caught be B, L and P respectively. Then $B = 3L = 4P$, so $L = \dfrac{B}{3}$ and $P = \dfrac{B}{4}$.

So

$$\begin{aligned} B+\frac{B}{3}+\frac{B}{4} &\leq 99 \\ \frac{19B}{12} &\leq 99 \\ B &\leq \frac{99\times 12}{19} \\ &\leq 12\times 5\frac{4}{19} = 60+2\frac{10}{19} \end{aligned}$$

and 60 is the largest number which is a multiple of 3 and 4,

hence (C).

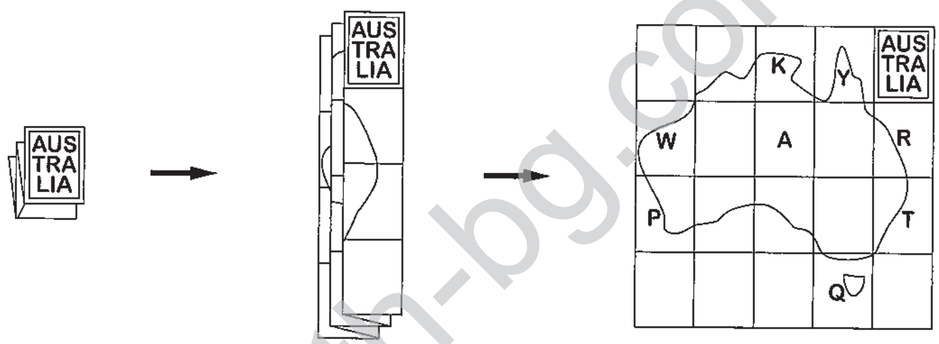
Alternative 2

Suppose Billy caught 12 fish. Then Lenny caught 4 and Peter caught 3. The total is 19. When 100 is divided by 19, the quotient is 5. Hence Billy caught $12\times 5 = 60$ fish,

hence (C).

23. (Also I21 & S20)

After refolding along vertical folds, the four panels are stacked, from top to bottom, YK, RAW, TP, Q.



After folding the horizontal folds, the second and fourth will be reversed giving YK, WAR, TP, Q,

hence (E).

24. The odd digits are 1, 3, 5, 7 and 9. For a number to be divisible by 3, the sum of its digits must be divisible by 3, regardless of their order.

Consider:

1, 3, 5, 7: sum is 16, not divisible by 3, so any order is not divisible by 3;
1, 3, 5, 9: sum is 18, so any order is divisible by 3;
1, 3, 7, 9: sum is 20, not divisible by 3, so any order is not divisible by 3;
1, 5, 7, 9: sum is 22, not divisible by 3, so any grouping is not divisible by 3;
3, 5, 7, 9: sum is 24, so any order is divisible by 3.

So $\frac{2}{5}$ are divisible by 3,

hence (D).

25. (Also I22 & S21)

Any palindromic number $xyyx$ can be written as
$1000x + 100y + 10y + x = 1001x + 110y$, where x and y are integers and $1 \leq x \leq 9$ and $0 \leq y \leq 9$.

Now $1001 = 7 \times 143$, so 1001 and every multiple of it is divisible by 7. There are nine such multiples 1001, 2002, 3003, ..., 9009.

110 is not divisible by 7, so $110y$ is not divisible by 7 unless y is divisible by 7, and this occurs when $y = 0$ (already dealt with above) or $y = 7$. This gives another nine palindromes, 1771, 2772, ..., 9779.

So there are $9 + 9 = 18$ such palindromes,

hence (D).

26. Given the subtraction,

$$\begin{array}{rcccc} & 4 & W & X & Y \\ - & Y & 5 & 3 & Z \\ \hline & 2 & 0 & 0 & 9 \\ \hline \end{array}$$

by looking at the left-hand digits, Y is 1 or 2. If Y is 1, then from the right-hand digits, Z is 2, X is 4, W is 5 and Y is 2, which is a contradiction.
If Y is 2, then Z is 3, X is 4 and W is 5, and this works.
So $W \times X \times Y \times Z = 5 \times 4 \times 2 \times 3 = 120$.

27. We are looking at the number of 3-digit numbers not containing any of the four digits 1, 2, 3 or 4.
This means that such a number can have any of the five digits 5, 6, 7, 8 and 9 as the hundreds digit, and any of the six digits 0, 5, 6, 7, 8 and 9 in the other two places.
So there are $5 \times 6 \times 6 = 180$ such numbers.

28. (Also I27 & S27)

The seven smallest ascending 3-digit numbers are
$n = 123, 124, 125, 126, 127, 128$ and 129.

$6 \times 123 = 738 \quad 6 \times 127 = 762$
$6 \times 124 = 744 \quad 6 \times 128 = 768$
$6 \times 125 = 750 \quad 6 \times 129 = 774$
$6 \times 126 = 756$

In none of these cases is $6n$ an ascending number.

Now, $6n$ must end with a 0, 2, 4, 6 or 8 and the sum of its digits must be divisible by 3.
124, 134 and 234 are the only ascending 3-digit numbers ending with 4 and $6n = 744, 804, 1404$ in these cases, none ascending.

Hence n ends in a 6 or an 8.

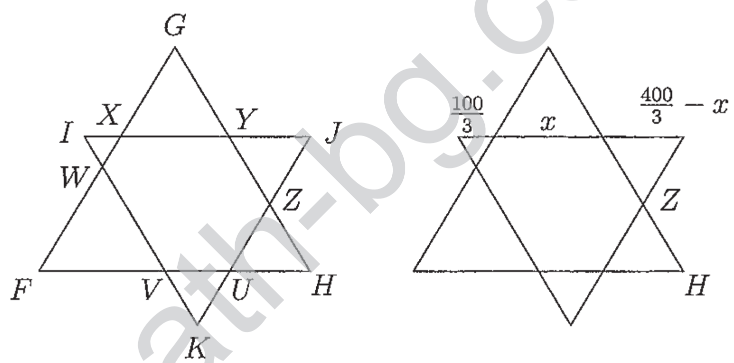
Consider those ending in 6.

n	$6n$	n	$6n$
136	816	146	876
156	936	236	1416
246	1476	256	1536
346	2076	356	2136
456	2736		

None of the $6n$ are ascending.

Consider then n ending with an 8, so is $ab8$ with $a < b < 8$. Then the carry into the tens digit of the product $6 \times n$ is 4. The next largest digit in n we can consider is $b = 7$ which gives $42 + 4$, so we get 6 in the tens digit of $6n$ and a carry of 4 to the hundreds digit of $6n$. This means the units digit of $6a$, must be less than 1 (otherwise $6n$ would not be ascending). Hence the first digit of n is 5, $6n$ is 3468 and n is 578.

29. All angles in the figure are 60° so all triangles in the figure are equilateral. $IX = XW = WI = \frac{100}{3}$, $IJ = JK = KI = \frac{500}{3}$ and $GH = HF = FG = \frac{700}{3}$. Let $XG = XY = YG = x$.



Then $YJ = YZ = \dfrac{500}{3} - x - \dfrac{100}{3} = \dfrac{400}{3} - x$.

Then $ZH = \dfrac{700}{3} - \left(\dfrac{400}{3} - x\right) - x = \dfrac{300}{3} = 100$, and the perimeter of $\triangle ZUH$ is $3 \times 100 = 300$.

30. (Also I28)

Alternative 1

Suppose Merlin starts with a rabbits and leaves b rabbits at each house. Then

place	number of rabbits
arrives house 1	$2a$
leaves house 1	$2a-b$
arrives house 2	$2(2a-b)=4a-2b$
leaves house 2	$4a-3b$
arrives house 3	$2(4a-3b)=8a-6b$
leaves house 3	$8a-7b$
arrives house 4	$2(8a-7b)=16a-14b$
leaves house 4	$16a-15b$
arrives house 5	$2(16a-15b)=32a-30b$
leaves house 5	$32a-31b$

So, if he leaves the last house with no rabbits, then $32a-31b=0$ and $32a=31b$. The smallest values of a and b are 31 and 32 respectively, so the minimum number of rabbits he could have at the start is 31.

Alternative 2

Assume that Merlin starts with x rabbits and leaves y rabbits at each of the houses. Then

$$\begin{aligned}((((2x-y)2-y)2-y)2-y)2-y &= 0\\ (((4x-3y)2-y)2-y)2-y &= 0\\ ((8x-7y)2-y)2-y &= 0\\ (16x-15y)2-y &= 0\\ 32x-31y &= 0\\ x &= \frac{31y}{32}\end{aligned}$$

The smallest value of y to make x an integer is $y=32$. So the minimum number of rabbits he has at the start is 31.

Generalisation

If there are n houses, the minimum number he could have at the start is 2^n-1.