

**LXII Национална олимпиада по математика – общински кръг
Гр. София, 15 декември 2012 г.**

12 клас - РЕШЕНИЯ И КРИТЕРИИ ЗА ОЦЕНЯВАНЕ

Задача 1.

$$\sin \alpha = 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}$$

.....

$$\sin \frac{\alpha}{2^{n-1}} = 2 \sin \frac{\alpha}{2^n} \cos \frac{\alpha}{2^n} \quad 2 \text{ т.}$$

$$\sin \alpha = 2^n \cos \frac{\alpha}{2} \cos \frac{\alpha}{2^2} \cos \frac{\alpha}{2^3} \dots \cos \frac{\alpha}{2^n} \sin \frac{\alpha}{2^n} \quad 1 \text{ т.}$$

$$\frac{\sin \alpha}{2^n \sin \frac{\alpha}{2^n}} = \cos \frac{\alpha}{2} \cos \frac{\alpha}{2^2} \cos \frac{\alpha}{2^3} \dots \cos \frac{\alpha}{2^n} \quad 1 \text{ т.}$$

$$\lim_{n \rightarrow \infty} 2^n \sin \frac{\alpha}{2^n} = \alpha \quad 2 \text{ т.}$$

$$\lim_{n \rightarrow \infty} \cos \frac{\alpha}{2} \cos \frac{\alpha}{2^2} \cos \frac{\alpha}{2^3} \dots \cos \frac{\alpha}{2^n} = \frac{\sin \alpha}{\alpha} \quad 1 \text{ т.}$$

Задача 2.

$$y' = (x-a)^2 + 2(x-a)$$

$$x_1 = a, x_2 = a-2 \quad 2 \text{ т.}$$

1 сл.) $a > 2$, $x_1 > 2$, $x_2 > 0$ - критичните точки $\notin [-2; 0]$,

$$\text{най-голяма стойност при } x=0 \quad y_{\text{НГС}} = y(0) = \frac{-a^3 + 3a^2}{3} \quad 2 \text{ т.}$$

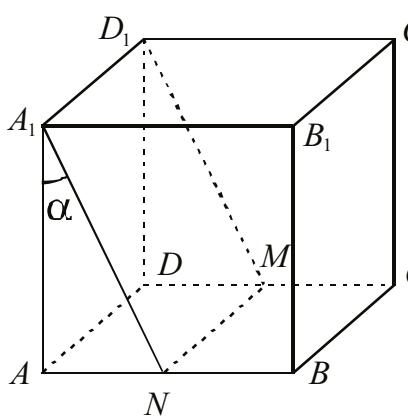
2 сл.) $0 < a \leq 2$

$$y_{\text{НГС}} = y(a-2) = \frac{4}{3} \quad 2 \text{ т.}$$

$$\text{Извод: При } a > 2 \quad y_{\text{НГС}} = \frac{-a^3 + 3a^2}{3}$$

$$\text{При } a \in (0; 2] \quad y_{\text{НГС}} = \frac{4}{3} \quad 1 \text{ т.}$$

Задача 3.



За обоснован чертеж и определяне вида на 2 части

$$AB = x \quad S = xy$$

$$A_1N = y \Rightarrow x = y \cos \alpha$$

$$AN = z \quad z = y \sin \alpha$$

$$x = \sqrt{m \cos \alpha}, \quad y = \sqrt{\frac{m}{\cos \alpha}}, \quad z = \sin \alpha \sqrt{\frac{m}{\cos \alpha}} \quad 2 \text{ т.}$$

$$\text{Триъгълна призма } V_1 = \frac{\sin \alpha}{2} \sqrt{m^3 \cos \alpha} \quad 1 \text{ т.}$$

$$\text{Четириъгълна призма } V_2 = x^3 - V_1 = \frac{2 \cos \alpha - \sin \alpha}{2} \sqrt{m^3 \cos \alpha} \quad 1 \text{ т.}$$

$$\frac{V_2}{V_1} = 2 \cot g \alpha - 1 \quad 2 \text{ т.}$$