Westpac

AUSTRALIAN MATHEMATICS COMPETITION
FOR THE WESTPAC AWARDS

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

WEDNESDAY 25 JULY 2007

INTERMEDIATE DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 9 AND 10
TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the **Answer Sheet** carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the Answer Sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMC reserves the right to re-examine students before deciding whether to grant official status to their score status to their score.

Intermediate Division

Questions 1 to 10, 3 marks each

1. $\dfrac{8 \times 9}{3}$ equals

(A) 27 (B) 11 (C) 24 (D) 20 (E) 17

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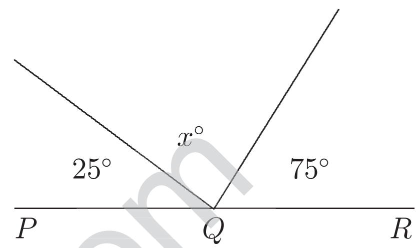
2. In the diagram, where PQR is a straight line, x equals

(A) 60 (B) 70 (C) 80 (D) 90 (E) 100



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3. If $110 + x = 97 + y$, then

(A) $x + 13 = y$ (B) $x = y + 13$ (C) $x + y = 13$
(D) $x + y = 207$ (E) $x - y = 207$

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4. Andy bought 2 chocolates at \$1.35 each. How much change should he get from \$5?

(A) \$2.70 (B) \$2.60 (C) \$3.30 (D) \$2 (E) \$2.30

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5. Of the following, which is the largest fraction?

(A) $\dfrac{7}{15}$ (B) $\dfrac{3}{7}$ (C) $\dfrac{6}{11}$ (D) $\dfrac{4}{9}$ (E) $\dfrac{1}{2}$

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6. The average weight of a group of 4 boys and 6 girls is 64 kg. The average weight of the boys is 70 kg. What is the average weight of the girls?

(A) 58 kg (B) 59 kg (C) 60 kg (D) 61 kg (E) 62 kg

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7. Nicky started a mobile phone call at 10:57 am. The charge for the call was 89 cents per minute and the total cost for the call was \$6.23. Nicky's call ended at

(A) 11:27 am (B) 11:14 am (C) 11:04 am (D) 11:46 am (E) 11:05 am

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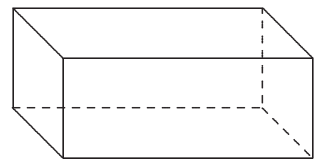
8. A fish tank in the shape of a rectangular prism is 1 m long and 25 cm wide. If it holds 55 L when full, its height, in centimetres, is



(A) 11 (B) 22 (C) 44 (D) 110 (E) 220

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9. The larger of two numbers is 3 more than twice the smaller number. If their sum is 18, what is the smaller number?

(A) 3 (B) 4 (C) 5 (D) 7 (E) 9

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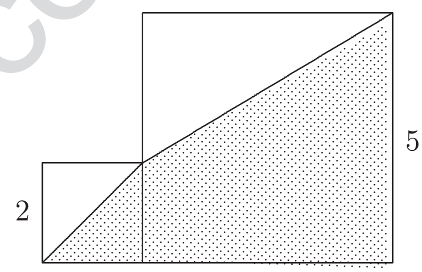
10. A square with side length 2 units is placed next to a square with side length 5 units as shown. The shaded area, in square units, is



(A) 13.5 (B) 14.5 (C) 18.5 (D) 19.5 (E) 26

Questions 11 to 20, 4 marks each

11. Successive discounts of 10%, 20% and 50% are equivalent to a single discount of

(A) 64% (B) 75% (C) $26\frac{2}{3}$% (D) 36% (E) 70%

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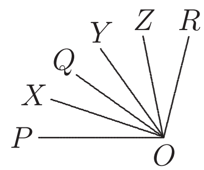
12. The game of *Four Tofu* is played on a 4×4 grid. When completed, each of the numbers 1, 2, 3 and 4 occurs in each row and column of the 4×4 grid and also in each 2×2 corner of the grid.
When the grid shown is completed, the sum of the four numbers in the corners of the 4×4 grid is

	2		
			1
	1	3	
4			

(A) 13 (B) 11 (C) 15 (D) 12 (E) 10

13. In the diagram, $\angle POX = \angle QOX$ and $\angle QOY = \angle YOZ = \angle ZOR$. If $\angle POY = 33°$ and $\angle XOZ = 45°$, the size of $\angle POR$ is



(A) 60° (B) 65° (C) 69° (D) 71° (E) 78°

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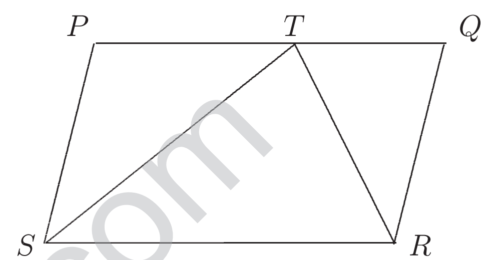
14. $PQRS$ is a parallelogram and T lies on PQ such that $PT : TQ = 3 : 2$. The ratio of the area of $PTRS$ to the area of $PQRS$ is

(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 4 : 5 (E) 5 : 6



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15. When 50 is divided by a whole number, the remainder is 5. How many different values are possible for this whole number?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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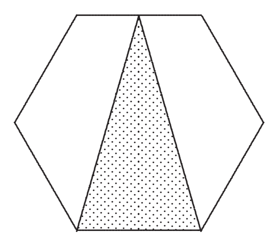
16. What fraction of the regular hexagon in the diagram is shaded?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{8}$ (D) $\frac{5}{12}$ (E) $\frac{1}{2}$



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17. In a school photo, the 630 pupils are arranged in rows. Each row has 3 more pupils in it than in the row in front. Of the numbers below, which number of rows is impossible?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

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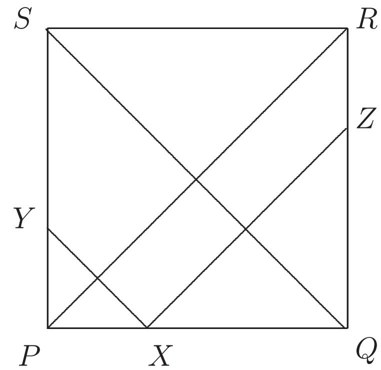
18. A number of runners competed in a race. When Jack finished, there were half as many runners who had finished before him compared to the number who finished behind him. Jill was the 10th runner to finish behind Jack and there were twice as many runners who had finished before her compared to the number who finished behind her. How many runners were there in the race?

(A) 27 (B) 28 (C) 29 (D) 30 (E) 31

19. $PQRS$ is a square with side $\sqrt{3}$. X is a point on PQ, Y and Z are points on PS and QR respectively. $XY \parallel QS$ and $XZ \parallel PR$. The sum of the lengths of XY and XZ is

(A) $\sqrt{5}$ (B) $\sqrt{6}$ (C) $\sqrt{7}$ (D) $\sqrt{8}$ (E) 3



20. Each of Andrew, Bill, Clair, Daniel and Eva either always lies or is always truthful, and they know which each of them is.

Andrew says that Bill is a liar.
Bill says that Clair is a liar.
Clair says that Daniel is a liar.
Daniel says that Eva is a liar.

The largest possible number of liars among them can be

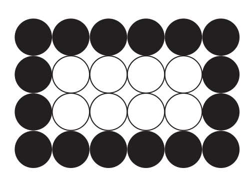
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Questions 21 to 25, 5 marks each

21. On her birthday in 2007, Rachel's age is equal to twice the sum of the digits of the year in which she was born. How many possible years are there in which she could have been born?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

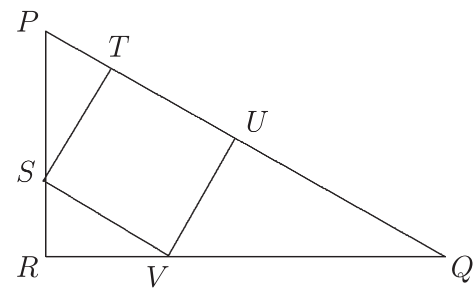
22. A border of black counters is placed around a rectangular array of white counters in a way similar to that shown in the diagram. If the number of white counters is equal to the number of black counters, for how many different numbers of white counters can this be done?



(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

23. PQR is a right-angled triangle with $PR = 3\,\text{cm}$ and $QR = 4\,\text{cm}$. The square $STUV$ is inscribed in $\triangle PQR$. What is the length, in centimetres, of the side of the square?

(A) $\frac{30}{17}$ (B) $\frac{12}{7}$ (C) $\frac{5}{3}$ (D) $\frac{60}{37}$ (E) $\frac{60}{39}$



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24. There are four lifts in a building. Each makes three stops, which do not have to be on consecutive floors or include the ground floor. For any two floors, there is at least one lift which stops on both of them. What is the maximum number of floors that this building can have?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 12

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25. A bee can fly or walk only in a straight line between any two corners on the inside of a cubic box of edge length 1. The bee managed to move so that it visited every corner of the box without passing through the same point twice in the air or on the wall of the box. The largest possible length of such a path is

(A) $2+5\sqrt{2}$ (B) $1+6\sqrt{2}$ (C) $7\sqrt{2}$ (D) $\sqrt{3}+6\sqrt{2}$ (E) $4\sqrt{3}+3\sqrt{2}$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. What is the smallest number of odd numbers in the range $1, \ldots, 2006$ such that, no matter how these numbers are chosen, there will always be two which add to 2008?

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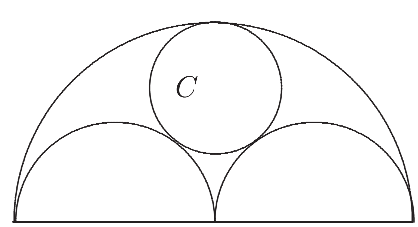
27. Two semicircles of radius 1 are drawn on the diameter of a semicircle of radius 2. A circle C touches all three semicircles as shown. If the radius of the circle C is $\frac{a}{b}$, where a are b are integers with no common factors, then what is the value of $a+b$?



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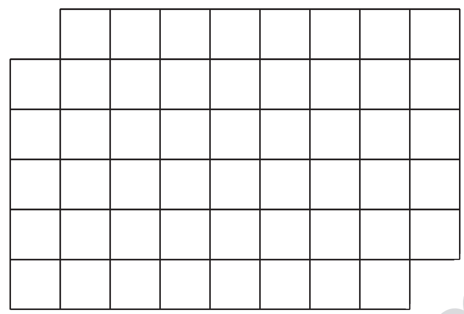
28. A *lucky number* is a positive integer which is 19 times the sum of its digits. How many different lucky numbers are there?

29. A grid of squares measuring 9 units by 6 units has the two corners removed as shown:



How many squares of any size are contained within this grid?

30. On my calculator screen the number 2659 can be read upside down as 6592. The digits that can be read upside down are 0, 1, 2, 5, 6, 8, 9 and are read as 0, 1, 2, 5, 9, 8, 6 respectively. Starting with 1, the fifth number that can be read upside down is 8 and the fifteenth is 21. What are the last three digits of the 2007th number that can be read upside down?

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