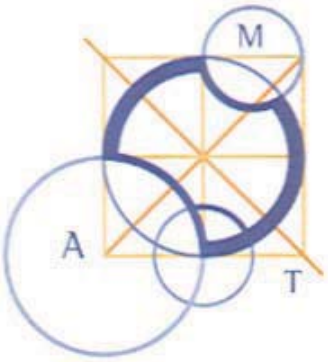
Westpac

AUSTRALIAN MATHEMATICS COMPETITION
FOR THE WESTPAC AWARDS

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST



THURSDAY 4 AUGUST 2005

INTERMEDIATE DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 9 AND 10
TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, rulers and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple choice questions, each with 5 possible answers given and 5 questions that require a whole number between 0 and 999. The questions generally get harder as you work through the paper.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the **Answer Sheet** carefully. Ensure your name, division, school and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only B or 2B lead pencil.
2. Record your answers on the reverse of the Answer Sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places so please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMC reserves the right to re-examine students before deciding whether to grant official status to their score.

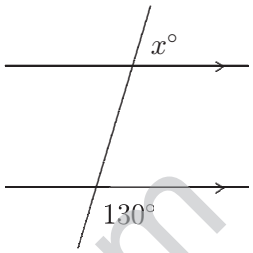
Intermediate Division

Questions 1 to 10, 3 marks each

1. The value of $2005 + 5002$ is

(A) 3003 (B) 4004 (C) 5555 (D) 2222 (E) 7007

2. In the diagram, the value of x is



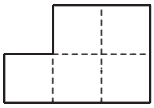
(A) 130 (B) 50 (C) 80 (D) 70 (E) 100

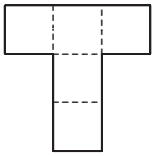
3. A lesson finished at 10:10 am. If the duration of the lesson was 55 minutes, it started at

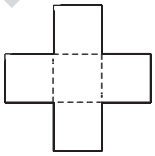
(A) 9:15 am (B) 9:45 am (C) 9:00 am (D) 8:45 am (E) 8:30 am

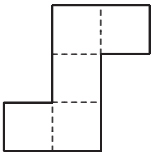
4. Which of these shapes has the smallest perimeter?

(A) 

(B) 

(C) 

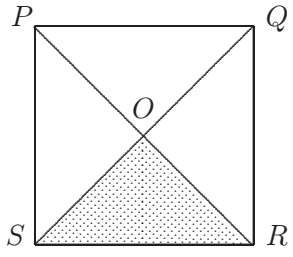
(D) 

(E) 

5. $1200 \div 40$ will have the same result as

(A) $600 \div 80$ (B) $2400 \div 20$ (C) $240 \div 8$ (D) $240 \div 5$ (E) $600 \div 8$

6. The diagonals of the square $PQRS$ intersect at O. The shaded region has area 16. What is the perimeter of the square?



(A) 4 (B) 8 (C) 16 (D) 32 (E) 64

I 2

7. $1 + \dfrac{1}{3 + \frac{1}{2}}$ equals

(A) $\dfrac{6}{5}$ (B) $\dfrac{7}{6}$ (C) $\dfrac{9}{2}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{9}{7}$

8. A two-digit number has tens digit t and its units digit u. If the digit 8 is placed between these digits, the value of the three-digit number is

(A) $t + u + 8$ (B) $10t + 80 + u$ (C) $10t + u + 8$
(D) $100t + 10u + 8$ (E) $100t + 80 + u$

9. The average of 6 numbers is 4.5. A further 2 numbers are added and the average is still 4.5. What is the sum of these two numbers?

(A) 27 (B) 9 (C) 36 (D) 4.5 (E) 8

10. Different tyres were fitted to a car, increasing the circumference of the wheels from 200 cm to 225 cm. On a journey of 1800 km, the number of revolutions of each wheel was reduced by

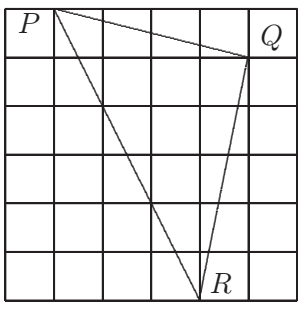
(A) 50 000 (B) 1000 (C) 2000 (D) 100 000 (E) 7 200 000

Questions 11 to 20, 4 marks each

11. Seven consecutive integers are listed. The sum of the smallest three is 33. What is the sum of the largest three?

(A) 39 (B) 37 (C) 42 (D) 48 (E) 45

12. The grid is a 1 cm grid. The area of the triangle PQR is



(A) 15 cm^2 (B) 10.5 cm^2 (C) 12 cm^2
(D) 13 cm^2 (E) 13.5 cm^2

I 3

13. When it is 12 noon in Montreal it is 6 pm in Paris. The times of take-off and landing of aircraft are given in local times. A plane leaving Montreal at 7 pm arrives in Paris at 8 am. Assuming that the travel time is the same in both directions, what time would a plane leaving Paris at 11 am arrive in Montreal?

(A) 12 noon (B) 6 pm (C) midnight (D) 11 am (E) 3 pm

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14. Two dice are thrown at random. The probability that the two numbers obtained are the two digits of a perfect square is

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{7}{36}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

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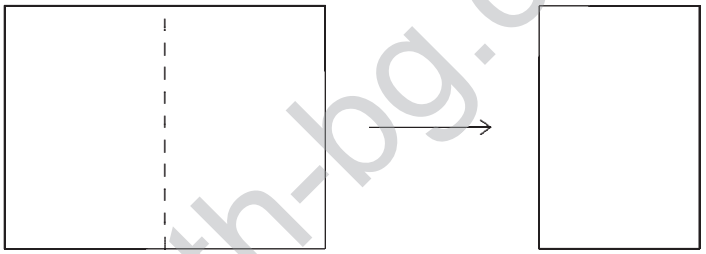
15. A rectangular sheet of cardboard is folded in half to form a smaller rectangle. The smaller rectangle is similar to the original rectangle. What is the ratio of the length to the width of the smaller rectangle?



(A) 2 : 1 (B) 3 : 2 (C) $\sqrt{3} : 1$ (D) $(1 + \sqrt{5}) : 2$ (E) $\sqrt{2} : 1$

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16. An aeroplane takes $2\frac{1}{2}$ hours to fly from Melbourne to Newcastle. If it were to increase its speed by 20%, how long would the trip take?

(A) 2 hours (B) 2 hours 5 minutes (C) 2 hours 10 minutes
(D) 2 hours 15 minutes (E) 2 hours 20 minutes

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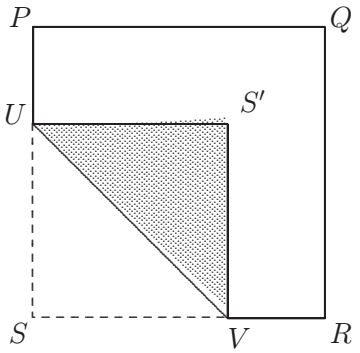
17. A square piece of paper has area 12 cm^2. It is coloured white on one side and shaded on the other. One corner of the paper has been folded over so that the sides of the triangle formed are parallel to the sides of the square as shown. The total visible area of the paper is half shaded and half white. What is the length, in centimetres, of the fold line UV?

(A) 4 (B) $\sqrt{12}$ (C) 3 (D) 6 (E) $\sqrt{8}$



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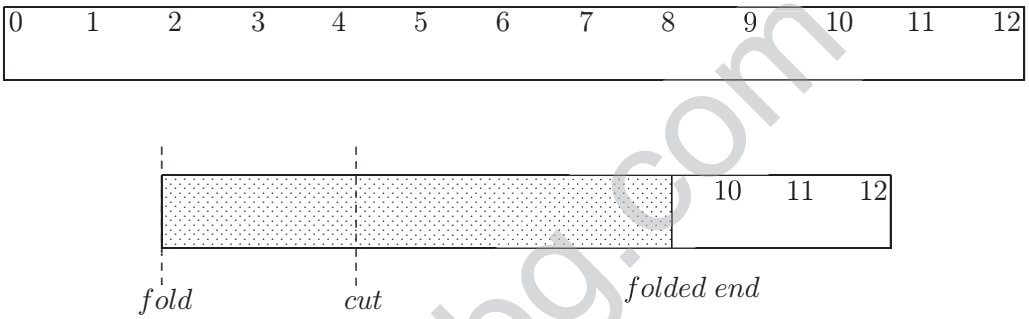
18. In the multiplication

$$\begin{array}{cccc} P & Q & R & \\ & & 3 & \times \\ \hline Q & Q & Q & \end{array}$$

each of P, Q and R represents a different digit. The sum of P, Q and R is

(A) 16 (B) 14 (C) 13 (D) 12 (E) 10

19. A 12 cm tape measure is folded back once on itself and a single cut is made through the folded tape, cutting it into 3 pieces.

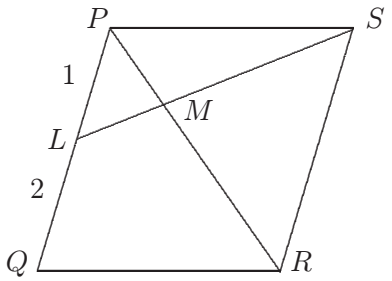


When the 3 resulting pieces are arranged from the shortest to the longest, the lengths are in the ratio 1 : 2 : 3. The number of places where the cut could be made is

(A) 0 (B) 1 (C) 2 (D) 4 (E) 6

20. $PQRS$ is a parallelogram and L is a point on the side PQ such that $PL = 1$ and $LQ = 2$. M is the point of intersection of PR and LS.
The ratio $PM : MR$ is equal to

(A) 1 : 3 (B) 1 : 4 (C) 1 : 2
(D) 2 : 5 (E) 2 : 7



Questions 21 to 30, 5 marks each

21. A train leaves Canberra for Sydney at 12 noon, and another train leaves Sydney for Canberra forty minutes later. Both trains follow the same route and travel at the same uniform speed, taking $3\frac{1}{2}$ hours to complete the journey. At what time will they pass?

(A) 1:45 pm (B) 2:00 pm (C) 2:05 pm (D) 2:15 pm (E) 2:25 pm

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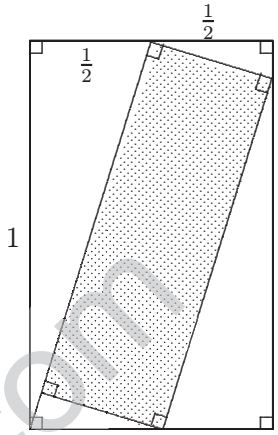
22. The number of 5-digit numbers in which every two neighbouring digits differ by 3 is

(A) 40 (B) 41 (C) 43 (D) 45 (E) 50

23. The area of the shaded rectangle is



(A) between $\frac{1}{4}$ and $\frac{5}{16}$

(B) between $\frac{5}{16}$ and $\frac{3}{8}$

(C) between $\frac{3}{8}$ and $\frac{7}{16}$

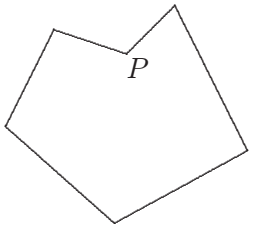
(D) between $\frac{7}{16}$ and $\frac{1}{2}$

(E) more than $\frac{1}{2}$

24. A 3×3 square is divided into nine 1×1 unit squares. Different integers from 1 to 9 are written into these nine unit squares. Consider the pairs of numbers in the squares sharing a common edge. What is the largest number of pairs where one number is a factor of the other number?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 12

25. The polygon shown has a reflex angle at P. In a polygon with n sides, what is the largest possible number of reflex angles?

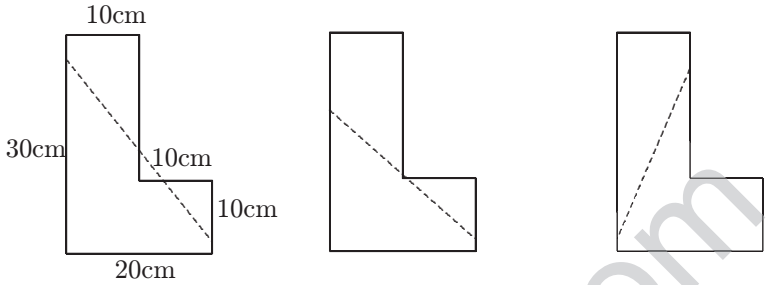


(A) 1 (B) 2 (C) $n-3$ (D) $n-2$ (E) $n-1$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

26. My name is Louis and my father has cooked me an L-shaped cake for my birthday. He says that I must cut it into three pieces with a single cut, so that my brother and sister can have a piece too. So, I have to cut it

like this or this but not like this.



He says that I have to be polite and let them have the first choice of the pieces, but I just know they'll be greedy and leave the smallest possible piece for me. So I want to cut the cake so that my little piece will be as big as possible. If I do this, how big, in square centimetres, will my piece be?

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27. A supermarket has seven checkout lanes. All seven checkouts accept cash payments but only lanes 1 to 4 allow credit cards. Kath, Kim and Sharon are all shopping and Kim insists on using her credit card while Kath and Sharon intend to pay cash. In how many ways could they choose their checkout lanes? (More than one could choose the same lane).

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28. Each point on the four sides of a 1 m $\times$ 1 m square is coloured one of n colours so that no two points that are exactly 1 m apart are coloured the same. What is the smallest n for which such a colouring can be made?

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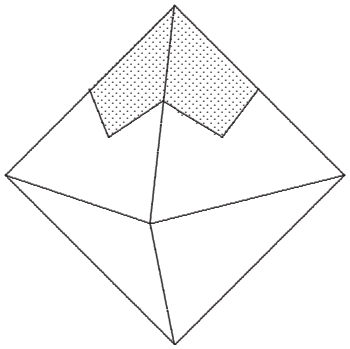
29. A regular octahedron has eight triangular faces and all sides the same length. A portion of a regular octahedron of volume 120 cm^3 consists of that part of it which is closer to the top vertex than to any other one. In the diagram, the outside part of this volume is shown shaded, and it extends down to the centre of the octahedron. What is the volume, in cubic centimetres, of this unusually shaped portion?



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30. A positive integer is equal to the sum of the squares of its four smallest positive divisors. What is the largest prime that divides this positive integer?
