AMC - 2007

Westpac

AUSTRALIAN MATHEMATICS COMPETITION FOR THE WESTPAC AWARDS

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

WEDNESDAY 25 JULY 2007

JUNIOR DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 7 AND 8

TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the **Answer Sheet** carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the Answer Sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMC reserves the right to re-examine students before deciding whether to grant official status to their score.

Junior Division

Questions 1 to 10, 3 marks each

1. The value of $37 - 16$ is

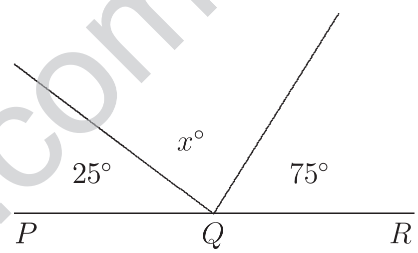
(A) 1 (B) 9 (C) 11 (D) 21 (E) -21

2. The value of $\dfrac{6 \times 7}{3}$ is

(A) 7 (B) 14 (C) 21 (D) 1 (E) 39

3. In the diagram, where PQR is a straight line, x equals

(A) 60 (B) 70 (C) 80 (D) 90 (E) 100



4. My mother left her car in the car park at 11 am and returned 5 hours later. The time when she returned was

(A) 6 pm (B) 5 pm (C) 4 pm (D) 3 pm (E) 2 pm

5. The sum of the smallest and largest of the five numbers 3.1, 0.6, 3, 6 and 0.5 is

(A) 6 (B) 9 (C) 6.1 (D) 6.6 (E) 6.5

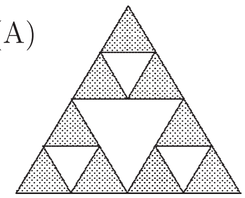
6. Tony wanted to add 97 to 298 and first added 100 to 300. What must he now subtract from this result to obtain $97 + 298$?

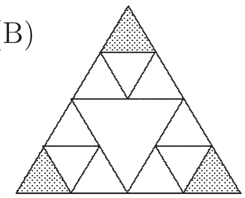
(A) 2 (B) 3 (C) 5 (D) 6 (E) 15

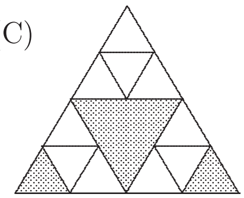
7. A tray of mangoes costs \$46. Each tray has 5 rows with 6 in each row. The cost of three mangoes is

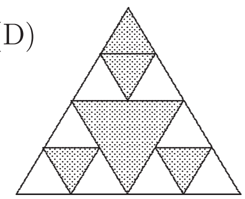
(A) \$4 (B) \$4.20 (C) \$4.30 (D) \$4.60 (E) \$5

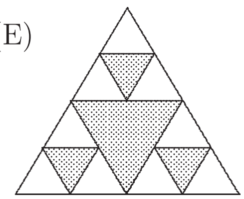
8. Which of the following shows three-eighths of the figure shaded?

(A) 

(B) 

(C) 

(D) 

(E) 

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9. If $97 + a = 100 + b$, then

(A) $a = b + 3$ (B) $a = b - 3$ (C) $a = 3b$ (D) $b = 3a$ (E) $a + 3 = b - 3$

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10. Of the following, which is the largest fraction?

(A) $\frac{7}{15}$ (B) $\frac{3}{7}$ (C) $\frac{6}{11}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

Questions 11 to 20, 4 marks each

11. Two cats together catch 60 mice. If Tiger catches three mice for every two Shorty catches, how many mice does Shorty catch?

(A) 20 (B) 24 (C) 30 (D) 36 (E) 40

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12. A class of 30 students has a spelling quiz every day. On Monday, 17 of the students scored 100% on the quiz. On Tuesday, 18 students scored 100% on the quiz. The least possible number of students who scored 100% on both quizzes is

(A) 1 (B) 5 (C) 13 (D) 15 (E) 17

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13. The game of *Four Tofu* is played on a 4×4 grid. When completed, each of the numbers 1, 2, 3 and 4 occurs in each row and column of the 4×4 grid and also in each 2×2 corner of the grid.
When the grid shown is completed, the sum of the four numbers in the corners of the 4×4 grid is

	2		
			1
	1	3	
4			

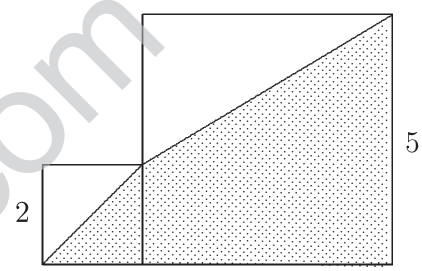
(A) 13 (B) 11 (C) 15 (D) 12 (E) 10

14. How many numbers in the range 100 to 1000 are divisible by 6?

(A) 136 (B) 150 (C) 160 (D) 165 (E) 166

15. A square with side length 2 units is placed next to a square with side length 5 units as shown. The shaded area, in square units, is

(A) 13.5 (B) 14.5 (C) 18.5 (D) 19.5 (E) 26

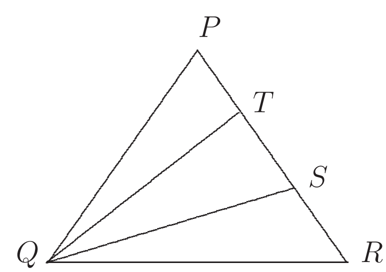


16. Ace, Bea, Cec, Dee, Eve, Fie and Geo are 1, 2, 3, 4, 5, 6 and 7 years old, in some order. Dee is three times as old as Bea. Cec is four years older than Eve. Fie is older than Ace and Ace is older than Geo, but the combined ages of Ace and Geo is greater than the age of Fie. The age of Ace is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

17. PQR is an equilateral triangle, QS and QT divide $\angle PQR$ into three equal parts. The size of $\angle QTS$, in degrees, is

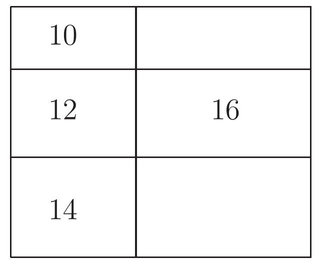
(A) 60 (B) 70 (C) 80 (D) 90 (E) 100



18. Jim's average score in his first six matches was 8.5. If all scores are whole numbers and his lowest score was 5, what is the lowest value which his highest score could have been?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

19. The diagram shows a large rectangle which has been divided into six smaller rectangles. Four of the smaller rectangles have *perimeters* of 10, 12, 14 and 16 centimetres, as shown by the numbers inside them.



If the sides of the rectangles are whole numbers, what is the smallest possible perimeter, in centimetres, of the large rectangle?

(A) 30 (B) 32 (C) 34 (D) 36 (E) 40

20. Jane's age is a prime number. Andy's age has 8 factors and he is one year older than Jane. Of the following numbers, which could be the sum of their ages?

(A) 27 (B) 39 (C) 75 (D) 87 (E) 107

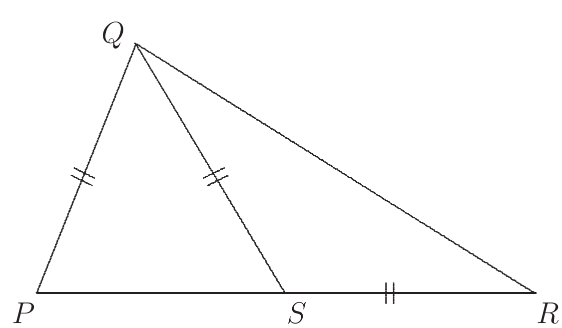
Questions 21 to 25, 5 marks each

21. On her birthday in 2007, Rachel's age is equal to twice the sum of the digits of the year in which she was born. How many possible years are there in which she could have been born?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

22. PQR is a triangle. S lies on PR such that $PQ = QS = SR$. All the angles in the diagram are a positive whole number of degrees. The largest possible size, in degrees, of $\angle PQR$ is

(A) 171 (B) 173 (C) 175 (D) 177 (E) 179



23. How many two-digit numbers are equal to three times the product of their digits?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

24. On a 3×5 chessboard, a counter can move one square at a time along a row or a column, but not along any diagonal. Starting from some squares, it can visit each of the other 14 squares exactly once, without returning to its starting square. Of the 15 squares, how many could be the counter's starting square?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

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25. Each of Andrew, Bill, Clair, Daniel and Eva either always lies or is always truthful, and they know which each of them is.

Andrew says that Bill is a liar.
Bill says that Clair is a liar.
Clair says that Daniel is a liar.
Daniel says that Eva is a liar.

The largest possible number of liars among them can be

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. Find the sum of all the two-digit integers, XY, between 10 and 99, which have the property that

$$\begin{array}{rcc} & X & Y \\ \times & X & Y \\ \hline \cdots & X & Y \\ \hline \end{array}$$

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27. A rectangular area measuring 3 units by 4 units on a wall is to be covered with 6 tiles each measuring 1 unit by 2 units. In how many ways can this be done?

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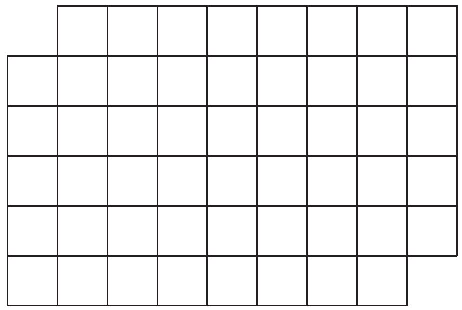
28. There are four lifts in a building. Each makes three stops, which do not have to be on consecutive floors or include the ground floor. For any two floors, there is at least one lift which stops on both of them. What is the maximum number of floors that this building can have?

29. A grid of squares measuring 9 units by 6 units has the two corners removed as shown:



How many squares of any size are contained within this grid?

30. For any positive integer N, consider the digits which occur either in N or in $7 \times N$. Let m be the smallest digit among those digits. What is the largest possible value of m?

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